

# Grades 9-12: Functions

Adopted 2019

## Functions

### A. Interpret functions that arise in applications in terms of the context. PC.F.A

24. Compare and contrast families of functions and their representations algebraically, graphically, numerically, and verbally in terms of their key features. Families of functions include but are not limited to linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, trigonometric, and their inverses. PC.F.A.24
25. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Extend from polynomial, exponential, logarithmic, and radical to rational and all trigonometric functions. PC.F.A.25
  - a. Find the difference quotient  $\frac{f(x + \Delta x) - f(x)}{\Delta x}$  of a function and use it to evaluate the average rate of change at a point. PC.F.A.25.A
  - b. Explore how the average rate of change of a function over an interval (presented symbolically or as a table) can be used to approximate the instantaneous rate of change at a point as the interval decreases. PC.F.A.25.B

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### B. Analyze functions using different representations. PC.F.B

26. Graph functions expressed symbolically and show key features of the graph, by hand and using technology. Use the equation of functions to identify key features in order to generate a graph. PC.F.B.26
  - a. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. PC.F.B.26.A
  - b. Graph trigonometric functions and their inverses, showing period, midline, amplitude, and phase shift. PC.F.B.26.B

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### C. Build a function that models a relationship between two quantities. PC.F.C

27. Compose functions. Extend to polynomial, trigonometric, radical, and rational functions. PC.F.C.27

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**D. Build new functions from existing functions.** PC.F.D

28. Find inverse functions. PC.F.D.28
- Given that a function has an inverse, write an expression for the inverse of the function. PC.F.D.28.A
  - Verify by composition that one function is the inverse of another. PC.F.D.28.B
  - Read values of an inverse function from a graph or a table, given that the function has an inverse. PC.F.D.28.C
  - Produce an invertible function from a non-invertible function by restricting the domain. PC.F.D.28.D
29. Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. Extend from logarithms with base 2 and 10 to a base of  $e$ . PC.F.D.29
30. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k \cdot f(x)$ ,  $f(k \cdot x)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Extend the analysis to include all trigonometric, rational, and general piecewise-defined functions with and without technology. PC.F.D.30
31. Graph conic sections from second-degree equations, extending from circles and parabolas to ellipses and hyperbolas, using technology to discover patterns. PC.F.D.31
- Graph conic sections given their standard form. PC.F.D.31.A
  - Identify the conic section that will be formed, given its equation in general form. PC.F.D.31.B

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**E. Recognize attributes of trigonometric functions and solve problems involving trigonometry.** PC.F.E

32. Solve application-based problems involving parametric and polar equations. PC.F.E.32
- Graph parametric and polar equations. PC.F.E.32.A
  - Convert parametric and polar equations to rectangular form. PC.F.E.32.B

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**F. Extend the domain of trigonometric functions using the unit circle.** PC.F.F

33. Use special triangles to determine geometrically the values of sine, cosine, and tangent for  $\pi/3$ ,  $\pi/4$ , and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number. PC.F.F.33
34. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. PC.F.F.34

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**G. Model periodic phenomena with trigonometric functions.** PC.F.G

35. Demonstrate that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. PC.F.G.35
36. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. PC.F.G.36

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**H. Prove and apply trigonometric identities.** PC.F.H

37. Use trigonometric identities to solve problems. PC.F.H.37
- a. Use the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  to derive the other forms of the identity. PC.F.H.37.A
- b. Use the angle sum formulas for sine, cosine, and tangent to derive the double angle formulas. PC.F.H.37.B
- c. Use the Pythagorean and double angle identities to prove other simple identities. PC.F.H.37.C