

# Math Plus

Adopted 2016

## Number and Quantity

### The Complex Number System

- A. Perform arithmetic operations with complex numbers. **P.N-CN.A**
  - 3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **P.N-CN.A.3**
- B. Represent complex numbers and their operations on the complex plane. **P.N-CN.B**
  - 4. Represent complex numbers on the complex plane in rectangular and polar form, including real and imaginary numbers, and explain why the rectangular and polar forms of a given complex number represent the same number. **P.N-CN.B.4**
  - 5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example,  $(-1 + \sqrt{3}i)^3 = 8$  because  $(-1 + \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ . **P.N-CN.B.5**
  - 6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **P.N-CN.B.6**
- C. Use complex numbers in polynomial identities and equations. **P.N-CN.C**
  - 8. Extend polynomial identities to the complex numbers. **P.N-CN.C.8**
  - 9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **P.N-CN.C.9**

## Vector and Matrix Quantities

- A. Represent and model with vector quantities. **P.N-VM.A**
1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. **P.N-VM.A.1**
  2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **P.N-VM.A.2**
  3. Solve problems involving velocity and other quantities that can be represented by vectors. **P.N-VM.A.3**
- B. Perform operations on vectors. **P.N-VN.B**
4. Add and subtract vectors. **P.N-VM.B.4**
    - a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **P.N-VM.B.4.A**
    - b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **P.N-VM.B.4.B**
    - c. Understand vector subtraction  $v - w$  as  $v + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **P.N-VM.B.4.C**
  5. Multiply a vector by a scalar. **P.N-VM.B.5**
    - a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise e.g., as  $c\langle v_x, v_y \rangle = \langle cv_x, cv_y \rangle$ . **P.N-VM.B.5.A**
    - b. Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ). **P.N-VM.B.5.B**
- C. Perform operations on matrices and use matrices in applications. **P.N-VM.C**
6. Use matrices to represent and manipulate data. **P.N-VM.C.6**
  7. Multiply matrices by scalars to produce new matrices. **P.N-VM.C.7**
  8. Add, subtract, and multiply matrices of appropriate dimensions. **P.N-VM.C.8**
  9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **P.N-VM.C.9**
  10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **P.N-VM.C.10**

11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **P.N-VM.C.11**
  12. Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area **P.N-VM.C.12**
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## Algebra

### Arithmetic with Polynomials and Rational Expressions

- C. Use polynomial identities to solve problems. **P.A-APR.C**
    5. Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument. **P.A-APR.C.5**
  - D. Rewrite rational expressions. **P.A-APR.D**
    7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **P.A-APR.D.7**
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### Reasoning with Equations and Inequalities

- C. Solve systems of equations. **P.A-REI.C**
    8. Represent a system of linear equations as a single matrix equation in a vector variable. **P.A-REI.C.8**
    9. Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater). **P.A-REI.C.9**
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## Functions

### Interpreting Functions

- C. Analyze functions using different representations. **P.F-IF.C**
  7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **P.F-IF.C.7**

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## Building Functions

- A. Build a function that models a relationship between two quantities. **P.F-BF.A**
  - 1. Write a function that describes a relationship between two quantities. **P.F-BF.A.1**
    - c. Compose functions. **P.F-BF.A.1.C**
- B. Build new functions from existing functions. **P.F-BF.B**
  - 4. Find inverse functions. **P.F-BF.B.4**
    - b. Verify by composition that one function is the inverse of another. **P.F-BF.B.4.B**
    - c. Read values of an inverse function from a graph or a table, given that the function has an inverse. **P.F-BF.B.4.C**
    - d. Produce an invertible function from a non-invertible function by restricting the domain. **P.F-BF.B.4.D**
  - 5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **P.F-BF.B.5**

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## Trigonometric Functions

- A. Extend the domain of trigonometric functions using the unit circle. **P.F-TF.A**
    - 3. Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number. **P.F-TF.A.3**
    - 4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **P.F-TF.A.4**
  - B. Model periodic phenomena with trigonometric functions. **P.F-TF.B**
    - 6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **P.F-TF.B.6**
    - 7. Use inverse functions to solve trigonometric equations utilizing real world context; evaluate the solution and interpret them in terms of context. **P.F-TF.B.7**
  - C. Apply trigonometric identities. **P.F-TF.C**
    - 9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **P.F-TF.C.9**
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## Geometry

### Similarity, Right Triangles, and Trigonometry

- D. Apply trigonometry to general triangles. **P.G-SRT.D**
    - 9. Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **P.G-SRT.D.9**
    - 10. Prove the Laws of Sines and Cosines and use them to solve problems. **P.G-SRT.D.10**
    - 11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **P.G-SRT.D.11**
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### Circles

- A. Understand and apply theorems about circles. **P.G-C.A**
    - 4. Construct a tangent line from a point outside a given circle to the circle. **P.G-C.A.4**
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### Expressing Geometric Properties with Equations

- A. Translate between the geometric description and the equation for a conic section. **P.G-GPE.A**
    - 2. Derive the equation of a parabola given a focus and directrix. **P.G-GPE.A.2**
    - 3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **P.G-GPE.A.3**
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### Geometric Measurement and Dimension

- A. Explain volume formulas and use them to solve problems. **P.G-GMD.A**
    - 2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **P.G-GMD.A.2**
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## Statistics and Probability

### Making Inferences and Justifying Conclusions

- B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. **P.S-IC.B**
  - 3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **P.S-IC.B.3**
  - 4. Use data from a random sample to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **P.S-IC.B.4**
  - 5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **P.S-IC.B.5**
  - 6. Evaluate reports based on data. **P.S-IC.B.6**

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## Conditional Probability and the Rules of Probability

- B. Use the rules of probability to compute probabilities of compound events in a uniform probability model. **P.S-CP.B**
  - 9. Use permutations and combinations to compute probabilities of compound events and solve problems. **P.S-CP.B.9**

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## Using Probability to Make Decisions

- A. Calculate expected values and use them to solve problems. **P.S-MD.A**
    - 1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **P.S-MD.A.1**
    - 2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **P.S-MD.A.2**
    - 3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated. Find the expected value. **P.S-MD.A.3**
    - 4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically. Find the expected value. **P.S-MD.A.4**
  - B. Use probability to evaluate outcomes of decisions. **P.S-MD.B**
    - 5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **P.S-MD.B.5**
      - a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. **P.S-MD.B.5.A**
      - b. Evaluate and compare strategies on the basis of expected values. **P.S-MD.B.5.B**
    - 6. Use randomization to make fair decisions based on probabilities. **P.S-MD.B.6**
    - 7. Analyze decisions and strategies using probability concepts. **P.S-MD.B.7**
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## Contemporary Mathematics

### Discrete Mathematics

- A. Understand and apply vertex-edge graph topics [P.CM-DM.A](#)
1. Study the following topics related to vertex-edge graph: Euler circuits, Hamilton circuits, shortest path, vertex coloring, and adjacency matrices. [P.CM-DM.A.1](#)
  2. Understand, analyze, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings. [P.CM-DM.A.2](#)
  3. Devise, analyze, and apply algorithms for solving vertex-edge graph problems. [P.CM-DM.A.3](#)
  4. Extend work with adjacency matrices for graphs, such as interpreting row sums and using the  $n$ th power of the adjacency matrix to count paths of length  $n$  in a graph. [P.CM-DM.A.4](#)
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## Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** [P.MP.1](#)

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2. **Reason abstractly and quantitatively.** [P.MP.2](#)

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3. **Construct viable arguments and critique the reasoning of others.** [P.MP.3](#)

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4. **Model with mathematics.** [P.MP.4](#)

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5. **Use appropriate tools strategically.** [P.MP.5](#)

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6. **Attend to precision.** [P.MP.6](#)

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7. **Look for and make use of structure.** [P.MP.7](#)

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8. **Look for and express regularity in repeated reasoning.** [P.MP.8](#)