

Grades 9-12: Standards

Number Sense and Operations

1 Generate equivalent expressions and perform operations with expressions involving exponents, radicals or logarithms.

- 1 Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents. [MA.912.NSO.1.1](#)
- 2 Generate equivalent algebraic expressions using the properties of exponents. [MA.912.NSO.1.2](#)
- 3 Generate equivalent algebraic expressions involving radicals or rational exponents using the properties of exponents. [MA.912.NSO.1.3](#)
- 4 Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals. [MA.912.NSO.1.4](#)
- 5 Add, subtract, multiply and divide algebraic expressions involving radicals. [MA.912.NSO.1.5](#)
- 6 Given a numerical logarithmic expression, evaluate and generate equivalent numerical expressions using the properties of logarithms or exponents. [MA.912.NSO.1.6](#)
- 7 Given an algebraic logarithmic expression, generate an equivalent algebraic expression using the properties of logarithms or exponents. [MA.912.NSO.1.7](#)

2 Represent and perform operations with expressions within the complex number system.

- 1 Extend previous understanding of the real number system to include the complex number system. Add, subtract, multiply and divide complex numbers. [MA.912.NSO.2.1](#)
- 2 Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane. [MA.912.NSO.2.2](#)
- 3 Calculate the distance and midpoint between two numbers on the complex coordinate plane. [MA.912.NSO.2.3](#)
- 4 Solve mathematical and real-world problems involving complex numbers represented algebraically or on the coordinate plane. [MA.912.NSO.2.4](#)
- 5 Represent complex numbers on the complex plane in rectangular and polar forms. [MA.912.NSO.2.5](#)
- 5 Rewrite complex numbers to trigonometric form. Multiply complex numbers in trigonometric form. [MA.912.NSO.2.6](#)

3 Represent and perform operations with vectors.

- 1 Apply appropriate notation and symbols to represent vectors in the plane as directed line segments. Determine the magnitude and direction of a vector in component form. [MA.912.NSO.3.1](#)
- 2 Represent vectors in component form, linear form or trigonometric form. Rewrite vectors from one form to another. [MA.912.NSO.3.2](#)
- 3 Solve mathematical and real-world problems involving velocity and other quantities that can be represented by vectors. [MA.912.NSO.3.3](#)
- 4 Solve mathematical and real-world problems involving vectors in two dimensions using the dot product and vector projections. [MA.912.NSO.3.4](#)
- 5 Solve mathematical and real-world problems involving vectors in three dimensions using the dot product and cross product. [MA.912.NSO.3.5](#)
- 6 Multiply a vector by a scalar algebraically or graphically. [MA.912.NSO.3.6](#)
- 7 Compute the magnitude and direction of a vector scalar multiple. [MA.912.NSO.3.7](#)
- 8 Add and subtract vectors algebraically or graphically. [MA.912.NSO.3.8](#)
- 9 Given the magnitude and direction of two or more vectors, determine the magnitude and direction of their sum. [MA.912.NSO.3.9](#)

4 Represent and perform operations with matrices.

- 1 Given a mathematical or real-world context, represent and manipulate data using matrices. [MA.912.NSO.4.1](#)
- 2 Given a mathematical or real-world context, represent and solve a system of two- or three-variable linear equations using matrices. [MA.912.NSO.4.2](#)
- 3 Solve mathematical and real-world problems involving addition, subtraction and multiplication of matrices. [MA.912.NSO.4.3](#)
- 4 Solve mathematical and real-world problems using the inverse and determinant of matrices. [MA.912.NSO.4.4](#)

Algebraic Reasoning

1 Interpret and rewrite algebraic expressions and equations in equivalent forms.

- 1 Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity. [MA.912.AR.1.1](#)
- 2 Rearrange equations or formulas to isolate a quantity of interest. [MA.912.AR.1.2](#)
- 3 Add, subtract and multiply polynomial expressions with rational number coefficients. [MA.912.AR.1.3](#)
- 4 Divide a polynomial expression by a monomial expression with rational number coefficients. [MA.912.AR.1.4](#)
- 5 Divide polynomial expressions using long division, synthetic division or algebraic manipulation. [MA.912.AR.1.5](#)

- 6 Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of polynomials. [MA.912.AR.1.6](#)
 - 7 Rewrite a polynomial expression as a product of polynomials over the real number system. [MA.912.AR.1.7](#)
 - 8 Rewrite a polynomial expression as a product of polynomials over the real or complex number system. [MA.912.AR.1.8](#)
 - 9 Apply previous understanding of rational number operations to add, subtract, multiply and divide rational algebraic expressions. [MA.912.AR.1.9](#)
 - 10 Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of rational algebraic expressions. [MA.912.AR.1.10](#)
 - 11 Apply the Binomial Theorem to create equivalent polynomial expressions. [MA.912.AR.1.11](#)
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2 Solve problems involving sequences and series.

- 1 Given a mathematical or real-world context, write and solve problems involving arithmetic sequences. [MA.912.AR.10.1](#)
 - 2 Given a mathematical or real-world context, write and solve problems involving geometric sequences. [MA.912.AR.10.2](#)
 - 3 Recognize and apply the formula for the sum of a finite arithmetic series to solve mathematical and real-world problems. [MA.912.AR.10.3](#)
 - 4 Recognize and apply the formula for the sum of a finite or an infinite geometric series to solve mathematical and real-world problems. [MA.912.AR.10.4](#)
 - 5 Given a mathematical or real-world context, write a sequence using function notation, defined explicitly or recursively, to represent relationships between quantities from a written description. [MA.912.AR.10.5](#)
 - 6 Given a mathematical or real-world context, find the domain of a given sequence defined recursively or explicitly. [MA.912.AR.10.6](#)
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3 Write, solve and graph linear equations, functions and inequalities in one and two variables.

- 1 Given a real-world context, write and solve one-variable multi-step linear equations. [MA.912.AR.2.1](#)
- 2 Write a linear two-variable equation to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context. [MA.912.AR.2.2](#)
- 3 Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point. [MA.912.AR.2.3](#)
- 4 Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features. [MA.912.AR.2.4](#)
- 5 Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.2.5](#)

- 6 Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically. [MA.912.AR.2.6](#)
 - 7 Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context. [MA.912.AR.2.7](#)
 - 8 Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality. [MA.912.AR.2.8](#)
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4 Write, solve and graph quadratic equations, functions and inequalities in one and two variables.

- 1 Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system. [MA.912.AR.3.1](#)
 - 2 Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real and complex number systems. [MA.912.AR.3.2](#)
 - 3 Given a mathematical or real-world context, write and solve one-variable quadratic inequalities over the real number system. Represent solutions algebraically or graphically. [MA.912.AR.3.3](#)
 - 4 Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context. [MA.912.AR.3.4](#)
 - 5 Given the x-intercepts and another point on the graph of a quadratic function, write the equation for the function. [MA.912.AR.3.5](#)
 - 6 Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context. [MA.912.AR.3.6](#)
 - 7 Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features. [MA.912.AR.3.7](#)
 - 8 Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.3.8](#)
 - 9 Given a mathematical or real-world context, write two-variable quadratic inequalities to represent relationships between quantities from a graph or a written description. [MA.912.AR.3.9](#)
 - 10 Given a mathematical or real-world context, graph the solution set to a two-variable quadratic inequality. [MA.912.AR.3.10](#)
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5 Write, solve and graph absolute value equations, functions and inequalities in one and two variables.

- 1 Given a mathematical or real-world context, write and solve one-variable absolute value equations. [MA.912.AR.4.1](#)
- 2 Given a mathematical or real-world context, write and solve one-variable absolute value inequalities. Represent solutions algebraically or graphically. [MA.912.AR.4.2](#)

- 3 Given a table, equation or written description of an absolute value function, graph that function and determine its key features. [MA.912.AR.4.3](#)
 - 4 Solve and graph mathematical and real-world problems that are modeled with absolute value functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.4.4](#)
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6 Write, solve and graph exponential and logarithmic equations and functions in one and two variables.

- 1 Solve one-variable exponential equations using the properties of exponents. [MA.912.AR.5.1](#)
 - 2 Solve one-variable equations involving logarithms or exponential expressions. Interpret solutions as viable in terms of the context and identify any extraneous solutions. [MA.912.AR.5.2](#)
 - 3 Given a mathematical or real-world context, classify an exponential function as representing growth or decay. [MA.912.AR.5.3](#)
 - 4 Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context. [MA.912.AR.5.4](#)
 - 5 Given an expression or equation representing an exponential function, reveal the constant percent rate of change per unit interval using the properties of exponents. Interpret the constant percent rate of change in terms of a real-world context. [MA.912.AR.5.5](#)
 - 6 Given a table, equation or written description of an exponential function, graph that function and determine its key features. [MA.912.AR.5.6](#)
 - 7 Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.5.7](#)
 - 8 Given a table, equation or written description of a logarithmic function, graph that function and determine its key features. [MA.912.AR.5.8](#)
 - 9 Solve and graph mathematical and real-world problems that are modeled with logarithmic functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.5.9](#)
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7 Solve and graph polynomial equations and functions in one and two variables.

- 1 Given a mathematical or real-world context, when suitable factorization is possible, solve one-variable polynomial equations of degree 3 or higher over the real and complex number systems. [MA.912.AR.6.1](#)
- 2 Explain and apply the Remainder Theorem to solve mathematical and real-world problems. [MA.912.AR.6.2](#)
- 3 Explain and apply theorems for polynomials to solve mathematical and real-world problems. [MA.912.AR.6.3](#)
- 4 Given a table, equation or written description of a polynomial function of degree 3 or higher, graph that function and determine its key features. [MA.912.AR.6.4](#)

- 5 Sketch a rough graph of a polynomial function of degree 3 or higher using zeros, multiplicity and knowledge of end behavior. [MA.912.AR.6.5](#)
 - 6 Solve and graph mathematical and real-world problems that are modeled with polynomial functions of degree 3 or higher. Interpret key features and determine constraints in terms of the context. [MA.912.AR.6.6](#)
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8 Solve and graph radical equations and functions in one and two variables.

- 1 Solve one-variable radical equations. Interpret solutions as viable in terms of context and identify any extraneous solutions. [MA.912.AR.7.1](#)
 - 2 Given a table, equation or written description of a square root or cube root function, graph that function and determine its key features. [MA.912.AR.7.2](#)
 - 3 Solve and graph mathematical and real-world problems that are modeled with square root or cube root functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.7.3](#)
 - 4 Solve and graph mathematical and real-world problems that are modeled with radical functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.7.4](#)
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9 Solve and graph rational equations and functions in one and two variables.

- 1 Write and solve one-variable rational equations. Interpret solutions as viable in terms of the context and identify any extraneous solutions. [MA.912.AR.8.1](#)
 - 2 Given a table, equation or written description of a rational function, graph that function and determine its key features. [MA.912.AR.8.2](#)
 - 3 Solve and graph mathematical and real-world problems that are modeled with rational functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.8.3](#)
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10 Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.

- 1 Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically. [MA.912.AR.9.1](#)
- 2 Given a mathematical or real-world context, solve a system consisting of a two-variable linear equation and a non-linear equation algebraically or graphically. [MA.912.AR.9.2](#)
- 3 Given a mathematical or real-world context, solve a system consisting of two-variable linear or non-linear equations algebraically or graphically. [MA.912.AR.9.3](#)
- 4 Graph the solution set of a system of two-variable linear inequalities. [MA.912.AR.9.4](#)
- 5 Graph the solution set of a system of two-variable inequalities. [MA.912.AR.9.5](#)
- 6 Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options. [MA.912.AR.9.6](#)

- 7 Given a real-world context, represent constraints as systems of linear and non-linear equations or inequalities. Interpret solutions to problems as viable or non-viable options. [MA.912.AR.9.7](#)
 - 8 Solve real-world problems involving linear programming in two variables. [MA.912.AR.9.8](#)
 - 9 Given a mathematical or real-world context, solve a system of three-variable linear equations algebraically. [MA.912.AR.9.9](#)
 - 10 Solve and graph mathematical and real-world problems that are modeled with piecewise functions. Interpret key features and determine constraints in terms of the context. [MA.912.AR.9.10](#)
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Functions

1 Understand, compare and analyze properties of functions.

- 1 Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it. [MA.912.F.1.1](#)
 - 2 Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output. [MA.912.F.1.2](#)
 - 3 Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval. [MA.912.F.1.3](#)
 - 4 Write an algebraic expression that represents the difference quotient of a function. Calculate the numerical value of the difference quotient at a given pair of points. [MA.912.F.1.4](#)
 - 5 Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions. [MA.912.F.1.5](#)
 - 6 Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions. [MA.912.F.1.6](#)
 - 7 Compare key features of two functions each represented algebraically, graphically, in tables or written descriptions. [MA.912.F.1.7](#)
 - 8 Determine whether a linear, quadratic or exponential function best models a given real-world situation. [MA.912.F.1.8](#)
 - 9 Determine whether a function is even, odd or neither when represented algebraically, graphically or in a table. [MA.912.F.1.9](#)
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2 Identify and describe the effects of transformations on functions. Create new functions given transformations.

- 1 Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$ and $f(x+k)$ for specific values of k . [MA.912.F.2.1](#)
- 2 Identify the effect on the graph of a given function of two or more transformations defined by adding a real number to the x - or y - values or multiplying the x - or y - values by a real number. [MA.912.F.2.2](#)

- 3 Given the graph or table of $f(x)$ and the graph or table of $f(x)+k$, $kf(x)$, $f(kx)$ and $f(x+k)$, state the type of transformation and find the value of the real number k . MA.912.F.2.3
 - 4 Given the graph or table of values of two or more transformations of a function, state the type of transformation and find the values of the real number that defines the transformation. MA.912.F.2.4
 - 5 Given a table, equation or graph that represents a function, create a corresponding table, equation or graph of the transformed function defined by adding a real number to the x - or y -values or multiplying the x - or y -values by a real number. MA.912.F.2.5
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3 Create new functions from existing functions.

- 1 Given a mathematical or real-world context, combine two functions, limited to linear and quadratic, using arithmetic operations. When appropriate, include domain restrictions for the new function. MA.912.F.3.1
 - 2 Given a mathematical or real-world context, combine two or more functions, limited to linear, quadratic, exponential and polynomial, using arithmetic operations. When appropriate, include domain restrictions for the new function. MA.912.F.3.2
 - 3 Solve mathematical and real-world problems involving functions that have been combined using arithmetic operations. MA.912.F.3.3
 - 4 Represent the composition of two functions algebraically or in a table. Determine the domain and range of the composite function. MA.912.F.3.4
 - 5 Solve mathematical and real-world problems involving composite functions. MA.912.F.3.5
 - 6 Determine whether an inverse function exists by analyzing tables, graphs and equations. MA.912.F.3.6
 - 7 Represent the inverse of a function algebraically, graphically or in a table. Use composition of functions to verify that one function is the inverse of the other. MA.912.F.3.7
 - 8 Produce an invertible function from a non-invertible function by restricting the domain. MA.912.F.3.8
 - 9 Solve mathematical and real-world problems involving inverse functions. MA.912.F.3.9
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Financial Literacy

1 Build mathematical foundations for financial literacy.

- 1 Extend previous knowledge of operations of fractions, percentages and decimals to solve real-world problems involving money and business. MA.912.FL.1.1
- 2 Extend previous knowledge of ratios and proportional relationships to solve real-world problems involving money and business. MA.912.FL.1.2
- 3 Solve real-world problems involving weighted averages using spreadsheets and other technology. MA.912.FL.1.3

2 Develop an understanding of basic accounting and economic principles.

- 1 Given assets and liabilities, calculate net worth using spreadsheets and other technology. [MA.912.FL.2.1](#)
- 2 Solve real-world problems involving profits, costs and revenues using spreadsheets and other technology. [MA.912.FL.2.2](#)
- 3 Explain how consumer price index (CPI), gross domestic product (GDP), stock indices, unemployment rate and trade deficit are calculated. Interpret their value in terms of the context. [MA.912.FL.2.3](#)
- 4 Given current exchange rates, convert between currencies. Solve real-world problems involving exchange rates. [MA.912.FL.2.4](#)
- 5 Develop budgets that fit within various incomes using spreadsheets and other technology. [MA.912.FL.2.5](#)
- 6 Given a real-world scenario, complete and calculate federal income tax using spreadsheets and other technology. [MA.912.FL.2.6](#)

3 Describe the advantages and disadvantages of short-term and long-term purchases.

- 1 Compare simple, compound and continuously compounded interest over time. [MA.912.FL.3.1](#)
- 2 Solve real-world problems involving simple, compound and continuously compounded interest. [MA.912.FL.3.2](#)
- 3 Solve real-world problems involving present value and future value of money [MA.912.FL.3.3](#)
- 4 Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth. [MA.912.FL.3.4](#)
- 5 Compare the advantages and disadvantages of using cash versus personal financing options. [MA.912.FL.3.5](#)
- 6 Calculate the finance charges and total amount due on a bill using various forms of credit using estimation, spreadsheets and other technology. [MA.912.FL.3.6](#)
- 7 Compare the advantages and disadvantages of different types of student loans by manipulating a variety of variables and calculating the total cost using spreadsheets and other technology. [MA.912.FL.3.7](#)
- 8 Calculate using spreadsheets and other technology the total cost of purchasing consumer durables over time given different monthly payments, down payments, financing options and fees. [MA.912.FL.3.8](#)
- 9 Compare the advantages and disadvantages of different types of mortgage loans by manipulating a variety of variables and calculating fees and total cost using spreadsheets and other technology. [MA.912.FL.3.9](#)

- 10 Analyze credit scores qualitatively. Explain how short-term and long-term purchases, including deferred payments, may increase or decrease credit scores. Explain how credit scores influence buying power. [MA.912.FL.3.10](#)
 - 11 Given a real-world scenario, establish a plan to pay off debt. [MA.912.FL.3.11](#)
 - 12 Given fixed costs, per item costs and selling price, determine the break-even point for sales volume. [MA.912.FL.3.12](#)
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4 Describe the advantages and disadvantages of financial and investment plans, including insurances.

- 1 Calculate and compare various options, deductibles and fees for various types of insurance policies using spreadsheets and other technology. [MA.912.FL.4.1](#)
 - 2 Compare the advantages and disadvantages for adding on a one-time warranty to a purchase using spreadsheets and other technology. [MA.912.FL.4.2](#)
 - 3 Compare the advantages and disadvantages of various retirement savings plans using spreadsheets and other technology. [MA.912.FL.4.3](#)
 - 4 Collect, organize and interpret data to determine an effective retirement savings plan to meet personal financial goals using spreadsheets and other technology. [MA.912.FL.4.4](#)
 - 5 Compare different ways that portfolios can be diversified in investments. [MA.912.FL.4.5](#)
 - 6 Simulate the purchase of a stock portfolio with a set amount of money, and evaluate its worth over time considering gains, losses and selling, taking into account any associated fees. [MA.912.FL.4.6](#)
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Geometric Reasoning

1 Prove and apply geometric theorems to solve problems.

- 1 Prove relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles. [MA.912.GR.1.1](#)
- 2 Prove triangle congruence or similarity using Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side, Angle-Angle and Hypotenuse-Leg. [MA.912.GR.1.2](#)
- 3 Prove relationships and theorems about triangles. Solve mathematical and real-world problems involving postulates, relationships and theorems of triangles. [MA.912.GR.1.3](#)
- 4 Prove relationships and theorems about parallelograms. Solve mathematical and real-world problems involving postulates, relationships and theorems of parallelograms. [MA.912.GR.1.4](#)
- 5 Prove relationships and theorems about trapezoids. Solve mathematical and real-world problems involving postulates, relationships and theorems of trapezoids. [MA.912.GR.1.5](#)
- 6 Solve mathematical and real-world problems involving congruence or similarity in two-dimensional figures. [MA.912.GR.1.6](#)

2 Apply properties of transformations to describe congruence or similarity.

- 1 Given a preimage and image, describe the transformation and represent the transformation algebraically using coordinates. [MA.912.GR.2.1](#)
- 2 Identify transformations that do or do not preserve distance. [MA.912.GR.2.2](#)
- 3 Identify a sequence of transformations that will map a given figure onto itself or onto another congruent or similar figure. [MA.912.GR.2.3](#)
- 4 Determine symmetries of reflection, symmetries of rotation and symmetries of translation of a geometric figure. [MA.912.GR.2.4](#)
- 5 Given a geometric figure and a sequence of transformations, draw the transformed figure on a coordinate plane. [MA.912.GR.2.5](#)
- 6 Apply rigid transformations to map one figure onto another to justify that the two figures are congruent. [MA.912.GR.2.6](#)
- 7 Justify the criteria for triangle congruence using the definition of congruence in terms of rigid transformations. [MA.912.GR.2.7](#)
- 8 Apply an appropriate transformation to map one figure onto another to justify that the two figures are similar. [MA.912.GR.2.8](#)
- 9 Justify the criteria for triangle similarity using the definition of similarity in terms of non-rigid transformations. [MA.912.GR.2.9](#)

3 Use coordinate geometry to solve problems or prove relationships.

- 1 Determine the weighted average of two or more points on a line. [MA.912.GR.3.1](#)
- 2 Given a mathematical context, use coordinate geometry to classify or justify definitions, properties and theorems involving circles, triangles or quadrilaterals. [MA.912.GR.3.2](#)
- 3 Use coordinate geometry to solve mathematical and real-world geometric problems involving lines, circles, triangles and quadrilaterals. [MA.912.GR.3.3](#)
- 4 Use coordinate geometry to solve mathematical and real-world problems on the coordinate plane involving perimeter or area of polygons. [MA.912.GR.3.4](#)

4 Use geometric measurement and dimensions to solve problems.

- 1 Identify the shapes of two-dimensional cross-sections of three-dimensional figures. [MA.912.GR.4.1](#)
- 2 Identify three-dimensional objects generated by rotations of two-dimensional figures. [MA.912.GR.4.2](#)
- 3 Extend previous understanding of scale drawings and scale factors to determine how dilations affect the area of two-dimensional figures and the surface area or volume of three-dimensional figures. [MA.912.GR.4.3](#)
- 4 Solve mathematical and real-world problems involving the area of two-dimensional figures. [MA.912.GR.4.4](#)

- 5 Solve mathematical and real-world problems involving the volume of three-dimensional figures limited to cylinders, pyramids, prisms, cones and spheres. [MA.912.GR.4.5](#)
 - 6 Solve mathematical and real-world problems involving the surface area of three-dimensional figures limited to cylinders, pyramids, prisms, cones and spheres. [MA.912.GR.4.6](#)
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5 Make formal geometric constructions with a variety of tools and methods.

- 1 Construct a copy of a segment or an angle. [MA.912.GR.5.1](#)
 - 2 Construct the bisector of a segment or an angle, including the perpendicular bisector of a line segment. [MA.912.GR.5.2](#)
 - 3 Construct the inscribed and circumscribed circles of a triangle. [MA.912.GR.5.3](#)
 - 4 Construct a regular polygon inscribed in a circle. Regular polygons are limited to triangles, quadrilaterals and hexagons. [MA.912.GR.5.4](#)
 - 5 Given a point outside a circle, construct a line tangent to the circle that passes through the given point. [MA.912.GR.5.5](#)
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6 Use properties and theorems related to circles.

- 1 Solve mathematical and real-world problems involving the length of a secant, tangent, segment or chord in a given circle. [MA.912.GR.6.1](#)
 - 2 Solve mathematical and real-world problems involving the measures of arcs and related angles. [MA.912.GR.6.2](#)
 - 3 Solve mathematical problems involving triangles and quadrilaterals inscribed in a circle. [MA.912.GR.6.3](#)
 - 4 Solve mathematical and real-world problems involving the arc length and area of a sector in a given circle. [MA.912.GR.6.4](#)
 - 5 Apply transformations to prove that all circles are similar. [MA.912.GR.6.5](#)
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7 Apply geometric and algebraic representations of conic sections.

- 1 Given a conic section, describe how it can result from the slicing of two cones. [MA.912.GR.7.1](#)
- 2 Given a mathematical or real-world context, derive and create the equation of a circle using key features. [MA.912.GR.7.2](#)
- 3 Graph and solve mathematical and real-world problems that are modeled with an equation of a circle. Determine and interpret key features in terms of the context. [MA.912.GR.7.3](#)
- 4 Given a mathematical or real-world context, derive and create the equation of a parabola using key features. [MA.912.GR.7.4](#)
- 5 Graph and solve mathematical and real-world problems that are modeled with an equation of a parabola. Determine and interpret key features in terms of the context. [MA.912.GR.7.5](#)

- 6 Given a mathematical or real-world context, derive and create the equation of an ellipse using key features. [MA.912.GR.7.6](#)
 - 7 Graph and solve mathematical and real-world problems that are modeled with an equation of an ellipse. Determine and interpret key features in terms of the context. [MA.912.GR.7.7](#)
 - 8 Given a mathematical or real-world context, derive and create the equation of a hyperbola using key features. [MA.912.GR.7.8](#)
 - 9 Graph and solve mathematical and real-world problems that are modeled with an equation of a hyperbola. Determine and interpret key features in terms of the context. [MA.912.GR.7.9](#)
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Data Analysis and Probability

1 Summarize, represent and interpret categorical and numerical data with one and two variables.

- 1 Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate. [MA.912.DP.1.1](#)
 - 2 Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display. [MA.912.DP.1.2](#)
 - 3 Explain the difference between correlation and causation in the contexts of both numerical and categorical data. [MA.912.DP.1.3](#)
 - 4 Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation. [MA.912.DP.1.4](#)
 - 5 Interpret the margin of error of a mean or percentage from a data set. Interpret the confidence level corresponding to the margin of error. [MA.912.DP.1.5](#)
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2 Solve problems involving univariate and bivariate numerical data.

- 1 For two or more sets of numerical univariate data, calculate and compare the appropriate measures of center and measures of variability, accounting for possible effects of outliers. Interpret any notable features of the shape of the data distribution. [MA.912.DP.2.1](#)
- 2 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. [MA.912.DP.2.2](#)
- 3 Estimate population percentages from data that has been fit to the normal distribution. [MA.912.DP.2.3](#)
- 4 Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data. [MA.912.DP.2.4](#)
- 5 Given a scatter plot that represents bivariate numerical data, assess the fit of a given linear function by plotting and analyzing residuals. [MA.912.DP.2.5](#)

- 6 Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context. [MA.912.DP.2.6](#)
 - 7 Compute the correlation coefficient of a linear model using technology. Interpret the strength and direction of the correlation coefficient. [MA.912.DP.2.7](#)
 - 8 Fit a quadratic function to bivariate numerical data that suggests a quadratic association and interpret any intercepts or the vertex of the model. Use the model to solve real-world problems in terms of the context of the data. [MA.912.DP.2.8](#)
 - 9 Fit an exponential function to bivariate numerical data that suggests an exponential association. Use the model to solve real-world problems in terms of the context of the data. [MA.912.DP.2.9](#)
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3 Solve problems involving categorical data.

- 1 Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context. [MA.912.DP.3.1](#)
 - 2 Given marginal and conditional relative frequencies, construct a two-way relative frequency table summarizing categorical bivariate data. [MA.912.DP.3.2](#)
 - 3 Given a two-way relative frequency table or segmented bar graph summarizing categorical bivariate data, interpret joint, marginal and conditional relative frequencies in terms of a real-world context. [MA.912.DP.3.3](#)
 - 4 Given a relative frequency table, construct and interpret a segmented bar graph. [MA.912.DP.3.4](#)
 - 5 Solve real-world problems involving univariate and bivariate categorical data. [MA.912.DP.3.5](#)
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4 Use and interpret independence and probability.

- 1 Describe events as subsets of a sample space using characteristics, or categories, of the outcomes, or as unions, intersections or complements of other events. [MA.912.DP.4.1](#)
- 2 Determine if events A and B are independent by calculating the product of their probabilities. [MA.912.DP.4.2](#)
- 3 Calculate the conditional probability of two events and interpret the result in terms of its context. [MA.912.DP.4.3](#)
- 4 Interpret the independence of two events using conditional probability. [MA.912.DP.4.4](#)
- 5 Given a two-way table containing data from a population, interpret the joint and marginal relative frequencies as empirical probabilities and the conditional relative frequencies as empirical conditional probabilities. Use those probabilities to determine whether characteristics in the population are approximately independent. [MA.912.DP.4.5](#)
- 6 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. [MA.912.DP.4.6](#)

- 7 Apply the addition rule for probability, taking into consideration whether the events are mutually exclusive, and interpret the result in terms of the model and its context. [MA.912.DP.4.7](#)
 - 8 Apply the general multiplication rule for probability, taking into consideration whether the events are independent, and interpret the result in terms of the context. [MA.912.DP.4.8](#)
 - 9 Apply the addition and multiplication rules for counting to solve mathematical and real-world problems, including problems involving probability. [MA.912.DP.4.9](#)
 - 10 Given a mathematical or real-world situation, calculate the appropriate permutation or combination. [MA.912.DP.4.10](#)
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5 Determine methods of data collection and make inferences from collected data.

- 1 Distinguish between a population parameter and a sample statistic. [MA.912.DP.5.1](#)
 - 2 Explain how random sampling produces data that is representative of a population. [MA.912.DP.5.2](#)
 - 3 Compare and contrast sampling methods. [MA.912.DP.5.3](#)
 - 4 Generate multiple samples or simulated samples of the same size to measure the variation in estimates or predictions. [MA.912.DP.5.4](#)
 - 5 Determine if a specific model is consistent within a given process by analyzing the data distribution from a data-generating process. [MA.912.DP.5.5](#)
 - 6 Determine the appropriate design, survey, experiment or observational study, based on the purpose. Articulate the types of questions appropriate for each type of design. [MA.912.DP.5.6](#)
 - 7 Compare and contrast surveys, experiments and observational studies. [MA.912.DP.5.7](#)
 - 8 Draw inferences about two populations using data and statistical analysis from two random samples. [MA.912.DP.5.8](#)
 - 9 Compare two treatments using data from an experiment in which the treatments are assigned randomly. [MA.912.DP.5.9](#)
 - 10 Determine whether differences between parameters are significant using simulations. [MA.912.DP.5.10](#)
 - 11 Evaluate reports based on data from diverse media, print and digital resources by interpreting graphs and tables; evaluating data-based arguments; determining whether a valid sampling method was used; or interpreting provided statistics. [MA.912.DP.5.11](#)
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6 Use probability distributions to solve problems.

- 1 Define a random variable for a quantity of interest by assigning a numerical value to each individual outcome in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. [MA.912.DP.6.1](#)

- 2 Develop a probability distribution for a discrete random variable using theoretical probabilities. Find the expected value and interpret it as the mean of the discrete distribution. [MA.912.DP.6.2](#)
 - 3 Develop a probability distribution for a discrete random variable using empirical probabilities. Find the expected value and interpret it as the mean of the discrete distribution. [MA.912.DP.6.3](#)
 - 4 Given a binomial distribution, calculate and interpret the expected value. Solve real-world problems involving binomial distributions. [MA.912.DP.6.4](#)
 - 5 Solve real-world problems involving geometric distributions. [MA.912.DP.6.5](#)
 - 6 Solve real-world problems involving Poisson distributions. [MA.912.DP.6.6](#)
 - 7 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values and standard deviations. Evaluate and compare strategies on the basis of the calculated expected values and standard deviations. [MA.912.DP.6.7](#)
 - 8 Apply probabilities to make fair decisions, such as drawing from lots or using a random number generator. [MA.912.DP.6.8](#)
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Trigonometry

1 Define and use trigonometric ratios, identities or functions to solve problems.

- 1 Define trigonometric ratios for acute angles in right triangles. [MA.912.T.1.1](#)
 - 2 Solve mathematical and real-world problems involving right triangles using trigonometric ratios and the Pythagorean Theorem. [MA.912.T.1.2](#)
 - 3 Apply the Law of Sines and the Law of Cosines to solve mathematical and real-world problems involving triangles. [MA.912.T.1.3](#)
 - 4 Solve mathematical problems involving finding the area of a triangle given two sides and the included angle. [MA.912.T.1.4](#)
 - 5 Prove Pythagorean Identities. Apply Pythagorean Identities to calculate trigonometric ratios and to solve problems. [MA.912.T.1.5](#)
 - 6 Prove the Double-Angle, Half-Angle, Angle Sum and Difference formulas for sine, cosine, and tangent. Apply these formulas to solve problems. [MA.912.T.1.6](#)
 - 7 Simplify expressions using trigonometric identities. [MA.912.T.1.7](#)
 - 8 Solve mathematical and real-world problems involving one-variable trigonometric ratios. [MA.912.T.1.8](#)
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2 Extend trigonometric functions to the unit circle.

- 1 Given any positive or negative angle measure in degrees or radians, identify its corresponding angle measure between 0° and 360° or between 0 and 2π . Convert between degrees and radians. [MA.912.T.2.1](#)
- 2 Define the six basic trigonometric functions for all real numbers by identifying corresponding angle measures and using right triangles drawn in the unit circle. [MA.912.T.2.2](#)
- 3 Determine the values of the six basic trigonometric functions for 0, $\pi/6$, $\pi/3$ and $\pi/4$ and their multiples using special triangles. [MA.912.T.2.3](#)

- 4 Use the unit circle to express the values of sine, cosine and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number. [MA.912.T.2.4](#)
- 5 Given angles measured in radians or degrees, calculate the values of the six basic trigonometric functions using the unit circle, trigonometric identities or technology. [MA.912.T.2.5](#)

3 Graph and apply trigonometric relations and functions.

- 1 Given a mathematical or real-world context, choose sine, cosine or tangent trigonometric functions to model periodic phenomena with specified amplitude, frequency, horizontal shift and midline. [MA.912.T.3.1](#)
- 2 Given a table, equation or written description of a trigonometric function, graph that function and determine key features. [MA.912.T.3.2](#)
- 3 Solve and graph mathematical and real-world problems that are modeled with trigonometric functions. Interpret key features and determine constraints in terms of the context. [MA.912.T.3.3](#)

4 Extend rectangular coordinates and equations to polar and parametric forms.

- 1 Define and plot polar coordinates. Convert between polar coordinates and rectangular coordinates with and without the use of technology. [MA.912.T.4.1](#)
- 2 Represent equations given in rectangular coordinates in terms of polar coordinates. Represent equations given in polar coordinates in terms of rectangular coordinates. [MA.912.T.4.2](#)
- 3 Graph equations in the polar coordinate plane with and without the use of graphing technology. [MA.912.T.4.3](#)
- 4 Identify and graph special polar equations, including circles, cardioids, limacons, rose curves and lemniscates. [MA.912.T.4.4](#)
- 5 Sketch the graph of a curve in the plane represented parametrically, indicating the direction of motion. [MA.912.T.4.5](#)
- 6 Convert from a parametric representation of a plane curve to a rectangular equation, and convert from a rectangular equation to a parametric representation of a plane curve. [MA.912.T.4.6](#)
- 7 Apply parametric equations to model applications involving motion in the plane. [MA.912.T.4.7](#)

Logic and Discrete Theory

1 Apply recursive methods to solve problems.

- 1 Apply recursive and iterative thinking to solve problems. [MA.912.LT.1.1](#)
- 2 Solve problems involving recurrence relations. [MA.912.LT.1.2](#)
- 3 Apply mathematical induction in a variety of applications. [MA.912.LT.1.3](#)

2 Apply optimization and techniques from Graph Theory to solve problems.

- 1 Define and explain the basic concepts of Graph Theory. [MA.912.LT.2.1](#)
- 2 Solve problems involving paths in graphs. [MA.912.LT.2.2](#)

- 3 Solve scheduling problems using critical path analysis and Gantt charts. Create a schedule using critical path analysis. [MA.912.LT.2.3](#)
 - 4 Apply graph coloring techniques to solve problems. [MA.912.LT.2.4](#)
 - 5 Apply spanning trees, rooted trees, binary trees and decision trees to solve problems. [MA.912.LT.2.5](#)
 - 6 Solve problems concerning optimizing resource usage using bin-packing techniques. [MA.912.LT.2.6](#)
 - 7 Solve problems involving optimal strategies in Game Theory. [MA.912.LT.2.7](#)
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3 Apply techniques from Election Theory and Fair Division Theory to solve problems.

- 1 Define and explain the basic concepts of Election Theory and voting. [MA.912.LT.3.1](#)
 - 2 Analyze election data using election theory techniques. Explain how Arrow's Impossibility Theorem may be related to the fairness of the outcome of the election. [MA.912.LT.3.2](#)
 - 3 Decide voting power within a group using weighted voting techniques. Provide real-world examples of weighted voting and its pros and cons. [MA.912.LT.3.3](#)
 - 4 Solve problems using fair division and apportionment techniques. [MA.912.LT.3.4](#)
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4 Develop an understanding of the fundamentals of propositional logic, arguments and methods of proof.

- 1 Translate propositional statements into logical arguments using propositional variables and logical connectives. [MA.912.LT.4.1](#)
- 2 Determine truth values of simple and compound statements using truth tables. [MA.912.LT.4.2](#)
- 3 Identify and accurately interpret "if...then," "if and only if," "all" and "not" statements. Find the converse, inverse and contrapositive of a statement. [MA.912.LT.4.3](#)
- 4 Represent logic operations, such as AND, OR, NOT, NOR, and XOR, using logical symbolism to solve problems. [MA.912.LT.4.4](#)
- 5 Determine whether two propositions are logically equivalent. [MA.912.LT.4.5](#)
- 6 Apply methods of direct and indirect proof and determine whether a logical argument is valid. [MA.912.LT.4.6](#)
- 7 Identify and give examples of undefined terms; axioms; theorems; proofs, including proofs using mathematical induction; and inductive and deductive reasoning. [MA.912.LT.4.7](#)
- 8 Construct proofs, including proofs by contradiction. [MA.912.LT.4.8](#)
- 9 Construct logical arguments using laws of detachment, syllogism, tautology, contradiction and Euler Diagrams. [MA.912.LT.4.9](#)
- 10 Judge the validity of arguments and give counterexamples to disprove statements. [MA.912.LT.4.10](#)

5 Apply properties from Set Theory to solve problems.

- 1 Given two sets, determine whether the two sets are equivalent and whether one set is a subset of another. Given one set, determine its power set. [MA.912.LT.5.1](#)
 - 2 Given a relation on two sets, determine whether the relation is a function, determine the inverse of the relation if it exists and identify if the relation is bijective. [MA.912.LT.5.2](#)
 - 3 Partition a set into disjoint subsets and determine an equivalence class given the equivalence relation on a set. [MA.912.LT.5.3](#)
 - 4 Perform the set operations of taking the complement of a set and the union, intersection, difference and product of two sets. [MA.912.LT.5.4](#)
 - 5 Explore relationships and patterns and make arguments about relationships between sets using Venn Diagrams. [MA.912.LT.5.5](#)
 - 6 Prove set relations, including DeMorgan's Laws and equivalence relations. [MA.912.LT.5.6](#)
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Calculus

1 Develop an understanding for limits and continuity. Determine limits and continuity.

- 1 Demonstrate understanding of the concept of a limit and estimate limits from graphs and tables of values. [MA.912.C.1.1](#)
 - 2 Determine the value of a limit if it exists algebraically using limits of sums, differences, products, quotients and compositions of continuous functions. [MA.912.C.1.2](#)
 - 3 Find limits of rational functions that are undefined at a point. [MA.912.C.1.3](#)
 - 4 Find one-sided limits. [MA.912.C.1.4](#)
 - 5 Find limits at infinity. [MA.912.C.1.5](#)
 - 6 Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. [MA.912.C.1.6](#)
 - 7 Find special limits by using the Squeeze Theorem or algebraic manipulation. [MA.912.C.1.7](#)
 - 8 Find limits of indeterminate forms using L'Hôpital's Rule. [MA.912.C.1.8](#)
 - 9 Define continuity in terms of limits. [MA.912.C.1.9](#)
 - 10 Given the graph of a function, identify whether a function is continuous at a point. If not, identify the type of discontinuity for the given function. [MA.912.C.1.10](#)
 - 11 Apply the Intermediate Value Theorem and the Extreme Value Theorem. [MA.912.C.1.11](#)
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2 Develop an understanding for and determine derivatives.

- 1 State, understand and apply the definition of derivative. Apply and interpret derivatives geometrically and numerically. [MA.912.C.2.1](#)

- 2 Interpret the derivative as an instantaneous rate of change or as the slope of the tangent line. [MA.912.C.2.2](#)
 - 3 Prove the rules for finding derivatives of constants, sums, products, quotients and the Chain Rule. [MA.912.C.2.3](#)
 - 4 Apply the rules for finding derivatives of constants, sums, products, quotients and the Chain Rule to solve problems with functions limited to algebraic, trigonometric, inverse trigonometric, logarithmic and exponential. [MA.912.C.2.4](#)
 - 5 Find the derivatives of implicitly defined functions. [MA.912.C.2.5](#)
 - 6 Find derivatives of inverse functions. [MA.912.C.2.6](#)
 - 7 Find second derivatives and derivatives of higher order. [MA.912.C.2.7](#)
 - 8 Find derivatives using logarithmic differentiation. [MA.912.C.2.8](#)
 - 9 Demonstrate and use the relationship between differentiability and continuity. [MA.912.C.2.9](#)
 - 10 Apply the Mean Value Theorem. [MA.912.C.2.10](#)
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3 Apply derivatives to solve problems.

- 1 Find the slope of a curve at a point, including points at which there are vertical tangent lines. [MA.912.C.3.1](#)
 - 2 Find an equation for the tangent line to a curve at a point and use it to make local linear approximation. [MA.912.C.3.2](#)
 - 3 Determine where a function is decreasing and increasing using its derivative. [MA.912.C.3.3](#)
 - 4 Find local and absolute maximum and minimum points of a function. [MA.912.C.3.4](#)
 - 5 Determine the concavity and points of inflection of a function using its second derivative. [MA.912.C.3.5](#)
 - 6 Sketch graphs by using first and second derivatives. Compare the corresponding characteristics of the graphs of f , f' and f'' . [MA.912.C.3.6](#)
 - 7 Solve optimization problems using derivatives. [MA.912.C.3.7](#)
 - 8 Find average and instantaneous rates of change. Explain the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed and acceleration. [MA.912.C.3.8](#)
 - 9 Find the velocity and acceleration of a particle moving in a straight line. [MA.912.C.3.9](#)
 - 10 Model and solve problems involving rates of change, including related rates. [MA.912.C.3.10](#)
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4 Develop an understanding for and determine integrals.

- 1 Interpret a definite integral as a limit of Riemann sums. Calculate the values of Riemann sums over equal subdivisions using left, right and midpoint evaluation points. [MA.912.C.4.1](#)

- 2 Apply Riemann sums, the Trapezoidal Rule and technology to approximate definite integrals of functions represented algebraically, geometrically and by tables of values. MA.912.C.4.2
 - 3 Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval. MA.912.C.4.3
 - 4 Evaluate definite integrals by using the Fundamental Theorem of Calculus. MA.912.C.4.4
 - 5 Analyze function graphs by using derivative graphs and the Fundamental Theorem of Calculus. MA.912.C.4.5
 - 6 Evaluate or solve problems using the properties of definite integrals. MA.912.C.4.6
 - 7 Evaluate definite and indefinite integrals by using integration by substitution. MA.912.C.4.7
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5 Apply integrals to solve problems.

- 1 Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions and solving applications related to motion along a line. MA.912.C.5.1
- 2 Solve separable differential equations. MA.912.C.5.2
- 3 Solve differential equations of the form $dY/dt=kY$ as applied to growth and decay problems. MA.912.C.5.3
- 4 Display a graphic representation of the solution to a differential equation by using slope fields, and locate particular solutions to the equation. MA.912.C.5.4
- 5 Find the area between a curve and the x-axis or between two curves by using definite integrals. MA.912.C.5.5
- 6 Find the average value of a function over a closed interval by using definite integrals. MA.912.C.5.6
- 7 Find the volume of a figure with known cross-sectional area, including figures of revolution, by using definite integrals. MA.912.C.5.7