

Multivariable Calculus

Mathematical Practices

0 Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals. MVC.MP

0.1 Make sense of problems and persevere in solving them. MVC.MP.1

0.2 Reason abstractly and quantitatively. MVC.MP.2

0.3 Construct viable arguments and critique the reasoning of others. MVC.MP.3

0.4 Model with mathematics. MVC.MP.4

0.5 Use appropriate tools strategically. MVC.MP.5

0.6 Attend to precision. MVC.MP.6

0.7 Look for and make use of structure. MVC.MP.7

0.8 Look for and express regularity in repeated reasoning. MVC.MP.8

Mathematical Modeling

1 Apply mathematics to real-life situations; model real-life phenomena using mathematics. MVC.MM.1

1.1 Explain contextual, mathematical problems using a mathematical model. MVC.MM.1.1

1.2 Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts. MVC.MM.1.2

1.3 Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation. MVC.MM.1.3

1.4 Use various mathematical representations and structures with this information to represent and solve real-life problems. MVC.MM.1.4

Patterning & Algebraic Reasoning

2 Express spatial and functional relationships with vectors, functions, and analytic geometry in three dimensions, and use these relationships to solve contextual, mathematical problems. MVC.PAR.2

2.1 Represent equations of lines in space using vectors. MVC.PAR.2.1

2.2 Express the analytic geometry of three dimensions in terms of the dot product and cross product of vectors. MVC.PAR.2.2

2.3 Use a linear system of equations to determine whether two planes intersect in a single point or a line, or whether they do not intersect at all. MVC.PAR.2.3

- 2.4 Evaluate functions of two independent variables at a point in the plane. MVC.PAR.2.4
- 2.5 Graph the level curves of functions of two independent variables. MVC.PAR.2.5
- 2.6 Investigate the continuity of functions of two independent variables in terms of the limits of such functions as (x, y) approaches a given point in the plane. MVC.PAR.2.6
- 2.7 Determine points or regions of discontinuity of functions of two independent variables. MVC.PAR.2.7

Abstract & Quantitative Reasoning

3 Define, describe, and represent the differentiation of functions of two independent variables and differential vectors to solve contextual, mathematical problems and to explain real-life phenomena. MVC.AQR.3

- 3.1 Approximate the partial derivatives at a point of a function defined by a table of data. MVC.AQR.3.1
- 3.2 Find expressions for the first and second partial derivatives of a function. MVC.AQR.3.2
- 3.3 Use the total differential to approximate mathematical models. MVC.AQR.3.3
- 3.4 Represent the partial derivatives of a system of two functions in two variables using the Jacobian. MVC.AQR.3.4
- 3.5 Find the partial derivatives of the composition of functions using the general chain rule. MVC.AQR.3
- 3.6 Apply partial differentiation to problems of optimization, including problems requiring the use of the Lagrange multiplier. MVC.AQR.3.5
- 3.7 Find the family of solutions and the envelope of the family of solutions to differential equations, including Clairaut equations. MVC.AQR.3.6
- 3.8 Define and apply the gradient, the divergence, and the curl in terms of differential vector operations. MVC.AQR.3.7

4 Interpret integrals of functions of two independent variables and of vector functions to solve contextual, mathematical problems and to explain real-life phenomena. MVC.AQR.4

- 4.1 Integrate functions of the form $z = f(x, y)$ or $w = f(x, y, z)$ through various techniques. MVC.AQR.4.1
- 4.2 Use, evaluate, and interpret double and triple integrals in terms of volume and mass. MVC.AQR.4.2
- 4.3 Represent and evaluate integrals of vector functions as double and triple integrals. MVC.AQR.4.3
- 4.4 Apply line and surface integral to functions representing real-world phenomena. MVC.AQR.4.4
- 4.5 Solve first-order exact differential equations. MVC.AQR.4.5
- 4.6 Use Green's Theorem to evaluate line integrals in the plane; use Stokes' Theorem to evaluate line integrals in space. MVC.AQR.4.6

- 4.7 Determine whether a line integral is independent of path and use line integrals in context. [MVC.AQR.4.7](#)
- 4.8 Use Gauss' Divergence Theorem to evaluate surface integrals. [MVC.AQR.4.8](#)
- 4.9 Define and apply the gradient, the divergence, and the curl in terms of integrals of vector functions. [MVC.AQR.4.9](#)