Maryland Mathematics

Precalculus B

Number and Quantity N

DOMAIN THE COMPLEX NUMBER SYSTEM N.CN

Cluster Perform arithmetic operations with complex numbers. A

3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. N.CN.A.3

Clarifications/Examples:

Understand the connection between modulus of a complex number and the magnitude of a vector.

Cluster Represent Complex Numbers and their operations on the complex plane. B

4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. N.CN.B.4

Clarifications/Examples:

Refer to the wording of the standard.

5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.B.5

Clarifications/Examples:

Refer to the wording of the standard.

6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. N.CN.B.6

Clarifications/Examples:

This could be addressed in a unit on vectors or polar coordinates.

DOMAIN VECTOR AND MATRIX QUANTITIES N.VM

Cluster Represent and model with vector quantities. A

1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (**ee**. **gg**. $\mathbf{vv}$, $|\mathbf{vv}|$, $\|\mathbf{vv}\|$, vv) N.VM.A.1

Clarifications/Examples:

Understand vocabulary and notation associated with the study of vectors (magnitude, direction, scalar, components, unit vector, resultant force).

Write position vectors from initial and terminal points.

2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N.VM.A.2

Clarifications/Examples:

Analyze vectors in terms of their horizontal and vertical components.

3 Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.A.3

Clarifications/Examples:

Model situations involving multiple vectors.

Determine resultant vectors and interpret them in context.

Cluster Perform operations on vectors. B

4 Add and subtract vectors. N.VM.B.4

Clarifications/Examples:

Refer to the wording of the standard.

4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.B.4.A

Clarifications/Examples:

Add vectors symbolically and graphically.

4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.B.4.B

Clarifications/Examples:

Use right triangles to derive formulas for the direction and magnitude of a vector.

Determine whether a system is in equilibrium.

4.c Understand vector subtraction **vv** − **ww** as **vv** + (−**ww**), where −**ww** is the additive inverse of **ww**, with the same magnitude as **ww** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. `N.VM.B.4.C`

Clarifications/Examples:

Refer to the wording of the standard.

5 Multiply a vector by a scalar. `N.VM.B.5`

Clarifications/Examples:

Refer to the wording of the standard.

5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c v v cv cv ( ) ( ) xy x y , , = . `N.VM.B.5.A`

Clarifications/Examples:

Refer to the wording of the standard.

5.b Compute the magnitude of a scalar multiple *cc***vv** using ‖*cc***vv**‖ = |*cc*|**vv**. Compute the direction of *cc***vv** knowing that when |*cc*|**vv** ≠ 0, the direction of *cc***vv** is either along **vv** (for *cc* > 0) or against **vv** (for *cc* > 0). `N.VM.B.5.B`

Clarifications/Examples:

Refer to the wording of the standard.

5.c Determine the dot product of two vectors. `N.VM.B.5.C`

Clarifications/Examples:

Refer to the wording of the standard.

Cluster Perform operations on matrices and use matrices in applications. `C`

6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. `N.VM.C.6`

Clarifications/Examples:

Refer to the wording of the standard.

7 Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. `N.VM.C.7`

Clarifications/Examples:

Refer to the wording of the standard.

8 Add, subtract, and multiply matrices of appropriate dimensions. `N.VM.C.8`

Clarifications/Examples:

Refer to the wording of the standard.

9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. N.VM.C.9

Clarifications/Examples:

Refer to the wording of the standard.

10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. N.VM.C.10

Clarifications/Examples:

Refer to the wording of the standard.

11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. N.VM.C.11

Clarifications/Examples:

Refer to the wording of the standard.

12 Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. N.VM.C.12

Clarifications/Examples:

Include rotations of 90°, 180°, and 270°, and reflections across x-axis, y-axis, and over the line $yy = xx$.

DOMAIN OPERATIONS AND ALGEBRAIC THINKING N.OA

Cluster Write and interpret numerical expressions. A

1 Use the notation for the factorial of a non-negative integer, n!, to evaluate expressions. N.OA.A.1

Clarifications/Examples:

Refer to the wording of the standard.

Algebra A

DOMAIN SEEING STRUCTURE IN EXPRESSIONS A.SSE

Cluster Write expressions in equivalent forms to solve problems. B

4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. A.SSE.B.4

Clarifications/Examples:

Connect prior learning from Algebra 2 to the new learning in Precalculus.

Solve real world problems that involve finite series.

4.a Express the sums in a series using sigma notation. A.SSE.B.4.A

Clarifications/Examples:

Use the algebra rules for finite sums to evaluate expressions written using sigma notation

Find the partial sums of a series defined using sigma notation.

5 Determine the sum, if it exists, of an infinite geometric series. A.SSE.B.5

Clarifications/Examples:

Use the sum of an infinite geometric series to express a repeating decimal as a rational number.

Solve real world problems that involve infinite series.

Use the algebra rules for finite sums to evaluate expressions written using sigma notation

DOMAIN ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSION A.APR

Cluster Use polynomial identities to solve problems. C

5 Know and apply the Binomial Theorem for the expansion of $(xx + yy)nn$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. A.APR.C.5

Clarifications/Examples:

Limit to $nn < 6$.

Limit to binomials with variable coefficient of one or constants less than four.

Understand the connection between the Binomial Theorem and an infinite series.

Functions F

DOMAIN INTERPRETING FUNCTIONS F.IF

Cluster Understand the concept of function and use function notation. A

3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.IF.A.3

Clarifications/Examples:

Understand the similarities and differences between linear functions and arithmetic sequences.

Understand similarities and differences between exponential functions and geometric sequences.

Include sequences that do not have a simple defining equation.

Cluster Analyze functions using different representations. B

10 Describe the behavior of a sequence. F.IF.C.10

Clarifications/Examples:

Generate the terms of a sequence given a formula for the nth term of the sequence.

Use the graph of a sequence to intuitively determine if the sequence converges or diverges.

Determine if a sequence is increasing, decreasing, or monotonic.

DOMAIN **BUILDING FUNCTIONS** F.BF

Cluster Build a function that models a relationship between two quantities. C

1 Write a function that describes a relationship between two quantities. F.BF.A.1

Clarifications/Examples:

Understand the connection between the formula for the general term *aann* of a given sequence and the related function that describes the relationship between the term number of the sequence and the value of the term.

Graph and analyze the functions that represent a given sequence.

1.a Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.A.1.A

Clarifications/Examples:

Include a variety of sequences that are neither arithmetic nor geometric.

2 Write sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.BF.A.2

Clarifications/Examples:

Construct a formula for the general term *aann* of a given sequence.

Functions F

DOMAIN INTERPRETING FUNCTIONS F.IF

Cluster Understand the concept of function and use function notation. A

2a Understand the concept of limit of a function. F.IF.A.2A

Clarifications/Examples:

Understand that the value of a function at a given point may not be the same as the limit as the function approaches the given point.

Understand when limits fail to exist.

Cluster Analyze functions using different representations. C

8c Interpret the behavior of the graph of a function using the concept of limits. F.IF.C.8C

Clarifications/Examples:

Use limits to reveal asymptotic or unbounded behavior.

Use limits to analyze functions for intervals of continuity or points of discontinuity.

Identify different types of discontinuities.

8d Estimate limits algebraically, numerically and graphically. F.IF.C.8D

Clarifications/Examples:

Use various algebraic techniques for evaluating limits.

Use a table of values to estimate a limit.

Use graphs to estimate limits.

Find one-sided limits.

Understand and use properties of limits.

Algebra A

DOMAIN SEEING STRUCTURE IN EXPRESSIONS A.SSE

Cluster Interpret the structure of expressions. A

2.a Analyze the structure of the general form of a second degree equation, $\boldsymbol{AAxx22 + BBBBBB + CCyy22 + DDDD + EEEE + FF} = \mathbf{00}$, to identify the conic section represented by the equation. A.SSE.A.2.A

Clarifications/Examples:

Evaluate the discriminant, $BB2 - 4AAAA$, of the general form of a second degree equation, $AAxx2 + BBBBBB + CCyy2 + DDDD + EEEE + FF = 0$, to determine if the graph of the equation is a circle, an ellipse, a hyperbola or a parabola.

3.d Choose and produce an equivalent form of a second degree equation, $AAxx2 + CCyy2 + DDDD + EEEE + FF = 0$, to reveal and explain properties of the conic section represented by the equation. A.SSE.B.3.D

Clarifications/Examples:

Produce the standard form of a second degree equation given the general form.

Recognize how the coefficients of the terms transform the conic section.

3.e Translate between the standard and general form, $AAxx2 + CCyy2 + DDDD + EEEE + FF = 0$, of a second degree equation. A.SSE.B.3.E

Clarifications/Examples:

Identify key features of a conic section.

Recognize how the coefficients of the terms transform the conic section.

Geometry G

DOMAIN GEOMETRIC MEASUREMENT AND DIMENSION G.GMD

Cluster Visualize relationships between two-dimensional and three-dimensional objects. B

4a Identify the shapes of two-dimensional cross-sections of a right double cone. G.GMD.B.4A

Clarifications/Examples:

Include special cases of plane/cone intersections that result in degenerate conic sections.

DOMAIN EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS G.GPE

Cluster Translate between the geometric description and the equation for a conic section. A

1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.A.1

Clarifications/Examples:

Understand and apply the locus definition for the circle.

Connect the geometric definition of the circle to its algebraic equation.

Understand the concept of eccentricity of conic sections, and understand the eccentricity of the circle.

Use eccentricity to write equations of circles.

2 Derive the equation of a parabola given a focus and directrix. G.GPE.A.2

Clarifications/Examples:

Include parabolas that have a horizontal axis of symmetry and parabolas that have a vertical axis of symmetry.

Derive the standard form of a parabola, centered at the origin.

Understand and apply the locus definition for the parabola.

Connect the geometric definition of the parabola to its algebraic equation.

Understand the concept of eccentricity of conic sections, and understand the eccentricity of the parabola.

Use eccentricity to write equations of parabolas.

3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G.GPE.A.3

Clarifications/Examples:

Derive the standard form of an ellipse and a hyperbola, centered at the origin.

Understand and apply the locus definitions of the ellipse and the hyperbola.

Connect the geometric definitions of the ellipse and hyperbola to their algebraic equations.

Understand the concept of eccentricity of conic sections, and understand the eccentricity of the ellipse and the hyperbola.

Use eccentricity to write equations of ellipses and hyperbolas.

Parametric Equations P

DOMAIN INTERPRETING PARAMETRIC EQUATIONS P.IPE

Cluster Analyze parametric equations. A

1 Sketch the curve defined by parametric equations. P.IPE.A.1

Clarifications/Examples:

Indicate with an arrow on the curve the direction in which the curve is traced as t increases.

Describe the motion of a particle with position ( ) x y , as t varies in a given interval.

2 Use parametric equations to model and solve motion problems. P.IPE.A.2

Clarifications/Examples:

Refer to the wording of the standard.

DOMAIN CREATING EQUATIONS P.CED

Cluster Creating equations that describe numbers or relationships. A

1 Create a single equation, using rectangular coordinates, that is equivalent to a pair of parametric equations. P.CED.A.1

Clarifications/Examples:

Refer to the wording of the standard.

2 Given a data set, create a parametric equation and a single equation using rectangular coordinates to fit the data. P.CED.A.2

Clarifications/Examples:

Refer to the wording of the standard.

DOMAIN EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS G.GPE

Cluster Polar Coordinates. C

8 Understand the relationship between polar coordinates and Cartesian coordinates. G.GPE.C.8

Clarifications/Examples:

Understand that a single point on the polar coordinate plane has more than one set of polar coordinates that can be used to identify the location of the point.

9 Convert between polar and rectangular coordinates. G.GPE.C.9

Clarifications/Examples:

Refer to the wording of the standard.

10 Plot points on a polar coordinate grid. G.GPE.C.10

Clarifications/Examples:

Refer to the wording of the standard.

Cluster Polar Equations. D

11 Convert equations between polar and rectangular forms. G.GPE.D.11

Clarifications/Examples:

Refer to the wording of the standard.

12 Graph polar equations by hand and using technology. G.GPE.D.12

Clarifications/Examples:

Make connections between the structure of a polar equation and the shape of the corresponding graph.

Graphs could include circles, lines, rose curves, cardioids, lemniscates, limaçons and spirals.

13 Solve systems of polar equations. G.GPE.D.13

Clarifications/Examples:

Restrict solutions to $[0, 2\pi)$.