Michigan Mathematics

High School — Functions

Adopted 2010

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** CCSS.MATH.PRACTICE.MP1
2. **Reason abstractly and quantitatively.** CCSS.MATH.PRACTICE.MP2
3. **Construct viable arguments and critique the reasoning of others.** CCSS.MATH.PRACTICE.MP3
4. **Model with mathematics.** CCSS.MATH.PRACTICE.MP4
5. **Use appropriate tools strategically.** CCSS.MATH.PRACTICE.MP5
6. **Attend to precision.** CCSS.MATH.PRACTICE.MP6
7. **Look for and make use of structure.** CCSS.MATH.PRACTICE.MP7
8. **Look for and express regularity in repeated reasoning.** CCSS.MATH.PRACTICE.MP8

Interpreting Functions

A. Understand the concept of a function and use function notation CCSS.MATH.CONTENT.HSF-IF.A

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). CCSS.MATH.CONTENT.HSF-IF.A.1
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. CCSS.MATH.CONTENT.HSF-IF.A.2
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. CCSS.MATH.CONTENT.HSF-IF.A.3

B. Interpret functions that arise in applications in terms of the context CCSS.MATH.CONTENT.HSF-IF.B

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. CCSS.MATH.CONTENT.HSF-IF.B.4
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. CCSS.MATH.CONTENT.HSF-IF.B.5

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6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. CCSS.MATH.CONTENT.HSF-IF.B.6

C. Analyze functions using different representations CCSS.MATH.CONTENT.HSF-IF.C

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. CCSS.MATH.CONTENT.HSF-IF.C.7
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima. CCSS.MATH.CONTENT.HSF-IF.C.7A
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. CCSS.MATH.CONTENT.HSF-IF.C.7B
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. CCSS.MATH.CONTENT.HSF-IF.C.7C
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. CCSS.MATH.CONTENT.HSF-IF.C.7D
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. CCSS.MATH.CONTENT.HSF-IF.C.7E
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. CCSS.MATH.CONTENT.HSF-IF.C.8
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. CCSS.MATH.CONTENT.HSF-IF.C.8A
   b. Use the properties of exponents to interpret expressions for exponential functions. CCSS.MATH.CONTENT.HSF-IF.C.8B
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). CCSS.MATH.CONTENT.HSF-IF.C.9

Building Functions

A. Build a function that models a relationship between two quantities CCSS.MATH.CONTENT.HSF-BF.A

1. Write a function that describes a relationship between two quantities CCSS.MATH.CONTENT.HSF-BF.A.1
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context. CCSS.MATH.CONTENT.HSF-BF.A.1A
   b. Combine standard function types using arithmetic operations. CCSS.MATH.CONTENT.HSF-BF.A.1B
   c. (+) Compose functions. CCSS.MATH.CONTENT.HSF-BF.A.1C

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2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. CCSS.MATH.CONTENT.HSF-BF.A.2

B. Build new functions from existing functions CCSS.MATH.CONTENT.HSF-BF.B

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. CCSS.MATH.CONTENT.HSF-BF.B.3
4. Find inverse functions. CCSS.MATH.CONTENT.HSF-BF.B.4
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. CCSS.MATH.CONTENT.HSF-BF.B.4A
   b. (+) Verify by composition that one function is the inverse of another. CCSS.MATH.CONTENT.HSF-BF.B.4B
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. CCSS.MATH.CONTENT.HSF-BF.B.4C
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain. CCSS.MATH.CONTENT.HSF-BF.B.4D
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. CCSS.MATH.CONTENT.HSF-BF.B.5

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Linear, Quadratic, and Exponential Models

A. Construct and compare linear, quadratic, and exponential models and solve problems CCSS.MATH.CONTENT.HSF-LE.A

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. CCSS.MATH.CONTENT.HSF-LE.A.1
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. CCSS.MATH.CONTENT.HSF-LE.A.1A
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. CCSS.MATH.CONTENT.HSF-LE.A.1B
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. CCSS.MATH.CONTENT.HSF-LE.A.1C
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). CCSS.MATH.CONTENT.HSF-LE.A.2
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. CCSS.MATH.CONTENT.HSF-LE.A.3
4. For exponential models, express as a logarithm the solution to ab^{ct} = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. CCSS.MATH.CONTENT.HSF-LE.A.4

B. Interpret expressions for functions in terms of the situation they model CCSS.MATH.CONTENT.HSF-LE.B

5. Interpret the parameters in a linear or exponential function in terms of a context. CCSS.MATH.CONTENT.HSF-LE.B.5

Trigonometric Functions

A. Extend the domain of trigonometric functions using the unit circle CCSS.MATH.CONTENT.HSF-TF.A

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. CCSS.MATH.CONTENT.HSF-TF.A.1
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. CCSS.MATH.CONTENT.HSF-TF.A.2
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number. CCSS.MATH.CONTENT.HSF-TF.A.3
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. CCSS.MATH.CONTENT.HSF-TF.A.4

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Model periodic phenomena with trigonometric functions CCSS.MATH.CONTENT.HSF-TF.B

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. CCSS.MATH.CONTENT.HSF-TF.B.5
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. CCSS.MATH.CONTENT.HSF-TF.B.6
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. CCSS.MATH.CONTENT.HSF-TF.B.7

C. Prove and apply trigonometric identities CCSS.MATH.CONTENT.HSF-TF.C

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. CCSS.MATH.CONTENT.HSF-TF.C.8
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. CCSS.MATH.CONTENT.HSF-TF.C.9