Wyoming Mathematics

Grades 9, 10, 11, 12

Adopted 2021

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** MP.1
2. **Reason abstractly and quantitatively.** MP.2
3. **Construct viable arguments and critique the reasoning of others.** MP.3
4. **Model with mathematics.** MP.4
5. **Use appropriate tools strategically.** MP.5
6. **Attend to precision.** MP.6
7. **Look for and make use of structure.** MP.7
8. **Look for and express regularity in repeated reasoning.** MP.8

The Real Number System

A. **Extend the properties of exponents to rational exponents.** N.RN.A

1. Explain how the meaning of the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. N.RN.A.1

Ad. Prove, use, and explain the properties of rational exponents (which are an extension of the properties of integer exponents) and extend to real-world context. N.RN.A.1.AD

P. Explain and use the meaning of rational exponents in terms of properties of integer exponents and use proper notation for radicals in terms of rational exponents. N.RN.A.1.P

Ba. Use proper notation for radicals in terms of rational exponents, but is unable to explain the meaning. N.RN.A.1.BA

BeB. May be able to use proper notation and use structure for integer exponents only. N.RN.A.1.BEB

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. N.RN.A.2

Ad. Compare contexts where radical form is preferable to rational exponents, and vice versa. N.RN.A.2.AD

P. Rewrite expressions involving radicals and rational exponents, using the properties of exponents. N.RN.A.2.P

Ba. Identify equivalent forms of expressions involving rational exponents (but is not able to rewrite or find the product of multiple radical expressions). N.RN.A.2.BA

BeB. May be able to convert radical notation to rational exponent notation. N.RN.A.2.BEB

B. Use properties of rational and irrational numbers. N.RN.B

3. Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. N.RN.B.3

Ad. Generalize the rules for sum and product properties of rational and irrational numbers. N.RN.B.3.AD

P. Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. N.RN.B.3.P

Ba. The Basic student is able to:
Explain why the sum or product of rational numbers is rational. OR
That the sum of a rational number and an irrational number is irrational. OR
That the product of a nonzero rational number and an irrational number is irrational. N.RN.B.3.BA

BeB. May be able to explain why adding and multiplying two rational numbers results in a rational number. N.RN.B.3.BEB

Quantities

C. Reason quantitatively and use units to solve problems. N.Q.C

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; and choose and interpret the scale and the origin in graphs and data displays. N.Q.C.1

Ad. Explain or defend their use of units as a way to understand problems and to guide the solution of multi-step problems; to explain and/or defend their choice of units consistently in formulas; and/or to explain and/or defend their choice of the scale and the origin in graphs and data displays. N.Q.C.1.AD

P. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; and choose and interpret the scale and the origin in graphs and data displays. N.Q.C.1.P

Ba. Inconsistently uses units as a way to understand problems and/or to guide the solution of problems; chooses and/or interprets units inconsistently in formulas; and/or inconsistently chooses and/or interprets the scale and the origin in graphs and data displays. N.Q.C.1.BA

BeB. May need guidance to use units as a way to understand problems and to guide the solution of problems; to choose and interpret units in formulas; and/or to choose and interpret the scale and the origin in graphs and data displays. N.Q.C.1.BEB

2. Define appropriate quantities for the purpose of descriptive modeling. N.Q.C.2

Ad. Explain or defend their choice of quantities for the purpose of descriptive modeling. N.Q.C.2.AD

P. Define appropriate quantities for the purpose of descriptive modeling. N.Q.C.2.P

Ba. Inconsistently defines quantities for the purpose of descriptive modeling. N.Q.C.2.BA

BeB. May need guidance to define quantities for the purpose of descriptive modeling. N.Q.C.2.BEB

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N.Q.C.3

Ad. Explain or defend their choice of level of accuracy appropriate to limitations on measurement when reporting quantities. N.Q.C.3.AD

P. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N.Q.C.3.P

Ba. Inconsistently chooses a level of accuracy appropriate to limitations on measurement when reporting quantities. N.Q.C.3.BA

BeB. Needs guidance to choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N.Q.C.3.BEB

The Complex Number System

D. Perform arithmetic operations with complex numbers. N.CN.D

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. N.CN.D.1

Ad. Make generalizations about the powers of i to write complex numbers in the form a + b with a and b being real numbers. N.CN.D.1.AD

P. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + b with a and b real. N.CN.D.1.P

Ba. Incorrectly and/or inconsistently applies −1 for i and/or i^2. N.CN.D.1.BA

BeB. Limited understanding of $i^2 = -1$ and needs guidance to correctly use i and/or i^2. N.CN.D.1.BEB

2. Use the relation $i^2 = -1$ and the Commutative, Associative, and Distributive Properties to add, subtract, and multiply complex numbers. N.CN.D.2

Ad. Perform arithmetic operations (add, subtract, multiply, divide) with complex numbers to include other powers of i and a + bi. N.CN.D.2.AD

P. Uses the relation $i^2 = -1$ and the Commutative, Associative, and Distributive Properties to add, subtract, and multiply complex numbers. N.CN.D.2.P

Ba. Inconsistently uses the relation $i^2 = -1$ and the Commutative, Associative, and Distributive Properties to add, subtract, and multiply complex numbers. N.CN.D.2.BA

BeB. Inconsistently uses the relation $i^2 = -1$ and the Commutative and Associative Properties to add and subtract complex numbers. N.CN.D.2.BEB

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. N.CN.D.3

Ad. (+) Simplify complex number expressions that involve a quotient and at least one other operation. N.CN.D.3.AD

P. (+)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. N.CN.D.3.P

Ba. (+) Given the conjugate of a complex number, find the quotients of complex numbers. N.CN.D.3.BA

BeB. (+) Does not meet the basic performance level. N.CN.D.3.BEB

E. Represent complex numbers and their operations on the complex plane. N.CN.E

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. N.CN.E.4

Ad. (+) The Advanced student is able to:
Given a complex number in rectangular form, convert it to polar form. AND
Given a complex number in polar form, convert it to rectangular form. N.CN.E.4.AD

P. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. N.CN.E.4.P

Ba. (+) Represent complex numbers on the complex plane in rectangular form. N.CN.E.4.BA

BeB. (+) Does not meet the basic performance level. N.CN.E.4.BEB

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.E.5

Ad. (+) Compare and contrast the algebraic and geometric approaches to finding the nth root of all real numbers. N.CN.E.5.AD

P. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.E.5.P

Ba. (+) Represent addition and subtraction of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.E.5.BA

BeB. (+) Does not meet the basic performance level. N.CN.E.5.BEB

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. N.CN.E.6

Ad. (+) Compare and contrast how distance and midpoint between two numbers are represented on the Cartesian plane versus the complex plane. N.CN.E.6.AD

P. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. N.CN.E.6.P

Ba. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference. N.CN.E.6.BA

BeB. (+) Does not meet the basic performance level. N.CN.E.6.BEB

F. Use complex numbers in polynomial identities and equations. N.CN.F

7. Solve quadratic equations with real coefficients that have complex solutions. N.CN.F.7

Ad. The Advanced student is able to:
Write a quadratic equation with real coefficients in standard form, when given a complex solution (recognizing that complex solutions to quadratic equations come in conjugate pairs). OR Determine the relationship between the solutions, the discriminant, and the graph of a quadratic equation with real coefficients. N.CN.F.7.AD

P. Solve quadratic equations with real coefficients that have complex solutions. N.CN.F.7.P

Ba. Solve quadratic equations with real coefficients that have pure imaginary solutions. N.CN.F.7.BA

BeB. May be able to determine which graphs have real solutions and which graphs have complex solutions when given a series of graphical representations of quadratic equations with real coefficients. N.CN.F.7.BEB

8. (+) Extend polynomial identities to the complex numbers. N.CN.F.8

Ad. (+) The Advanced student is able to:
Explore patterns in polynomial identities that are expressed with complex numbers. OR
Compare and contrast how polynomial identities are expressed in the real and the complex number system. N.CN.F.8.AD

P. (+) Extend polynomial identities to the complex numbers. N.CN.F.8.P

Ba. (+) Match equivalent forms of polynomial identities expressed with complex numbers. N.CN.F.8.BA

BeB. (+) Does not meet the basic performance level. N.CN.F.8.BEB

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. N.CN.F.9

Ad. (+) Explore the solutions to quadratic polynomials with complex coefficients to identify patterns supporting the Fundamental Theorem of Algebra. N.CN.F.9.AD

P. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. N.CN.F.9.P

Ba. (+) Solve a given quadratic polynomial with real coefficients and discuss how the Fundamental Theorem of Algebra is validated. N.CN.F.9.BA

BeB. (+) Does not meet the basic performance level. N.CN.F.9.BEB

Vector And Matrix Quantities

G. Represent and model with vector quantities. N.VM.G

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). N.VM.G.1

Ad. (+) Create real-world examples in different units that are represented as vectors. [This is identified as having a cross-curricular connection to physics.] N.VM.G.1.AD

P. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g. v, |v|, ||v||, v). N.VM.G.1.P

Ba. (+) Identify quantities that could be represented with a vector when given a real-world example (e.g., I drove 10 mph versus I drove west at 10 mph). N.VM.G.1.BA

BeB. (+) Does not meet the basic performance level. N.VM.G.1.BEB

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N.VM.G.2

Ad. (+) Model a real-world situation where subtracting the initial and terminal points of a vector would be applied. N.VM.G.2.AD

P. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N.VM.G.2.P

Ba. (+) Identify the initial and terminal points when a vector is represented graphically. N.VM.G.2.BA

BeB. (+) Does not meet the basic performance level. N.VM.G.2.BEB

3. (+) Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.G.3

Ad. (+) Create and solve real-world problems involving velocity and other quantities that can be represented by vectors. N.VM.G.3.AD

P. (+) Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.G.3.P

Ba. (+) Solve problems involving velocity that can be represented by vectors. N.VM.G.3.BA

BeB. (+) Does not meet the basic performance level. N.VM.G.3.BEB

H. Perform operations on vectors. N.VM.H

4. (+) Add and subtract vectors. N.VM.H.4

   A. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.H.4.A

   B. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.H.4.B

   C. Understand vector subtraction v - w as v + (-w), where (-w) is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. N.VM.H.4.C

Ad. (+) Create a real-world problem that requires addition or subtraction of two or more vectors, find the resultant vector, and interpret the magnitude and direction of the resultant vector. N.VM.H.4.AD

P. (+) Add and subtract vectors. N.VM.H.4.P

A. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.H.4.P.A

B. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.H.4.P.B

C. Understand vector subtraction v – w as v + (-w), where (-w) is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. N.VM.H.4.P.C

Ba. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.H.4.BA

BeB. (+) Does not meet the basic performance level. N.VM.H.4.BEB

5. (+) Multiply a vector by a scalar. N.VM.H.5

A. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. N.VM.H.5.A

B. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when INSERT $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). N.VM.H.5.B

Ad. (+) Create a real-world problem that requires scalar multiplication, find the resultant vector, and interpret the magnitude and direction of the resultant vector. N.VM.H.5.AD

P. (+) Multiply a vector by a scalar. N.VM.H.5.P

A. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. N.VM.H.5.P.A

B. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). N.VM.H.5.P.B

Ba. (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. N.VM.H.5.BA

BeB. (+) Does not meet the basic performance level. N.VM.H.5.BEB

I. Perform operations on matrices and use matrices in applications. N.VM.I

6. (+) Use matrices to represent and manipulate data. N.VM.I.6

Ad. (+) Using data from a real-world situation, create a matrix, analyze the situation, and make decisions. N.VM.I.6.AD

P. (+) Use matrices to represent and manipulate data. N.VM.I.6.P

Ba. (+) State what the data represents for elements in a given matrix. N.VM.I.6.BA

BeB. (+) Does not meet the basic performance level. N.VM.I.6.BEB

7. (+) Multiply matrices by scalars to produce new matrices. N.VM.I.7

Ad. (+) In a real-world situation involving scalar multiplication, place data in a matrix, identify and interpret the scalar, and interpret the results. N.VM.I.7.AD

P. (+) Multiply matrices by scalars to produce new matrices. N.VM.I.7.P

Ba. (+) Multiply matrices by integer scalars to produce new matrices. N.VM.I.7.BA

BeB. (+) Does not meet the basic performance level. N.VM.I.7.BEB

8. (+) Add, subtract, and multiply matrices of appropriate dimensions. N.VM.I.8

Ad. (+) Apply addition, subtraction, and/or multiplication of matrices to real-world situations. N.VM.I.8.AD

P. (+) Add, subtract, and multiply matrices of appropriate dimensions. N.VM.I.8.P

Ba. (+) Add and subtract matrices of dimensions limited to m and n less than or equal to 3. N.VM.I.8.BA

BeB. (+) Does not meet the basic performance level. N.VM.I.8.BEB

9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the Associative and Distributive Properties. N.VM.I.9

Ad. (+) Compare and contrast the Commutative, Associative, and Distributive Properties for the addition, subtraction, and multiplication of non-square matrices versus the addition, subtraction, and multiplication of real-numbers. N.VM.I.9.AD

P. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the Associative and Distributive Properties. N.VM.I.9.P

Ba. (+) Calculate AB, BA, (AB)C, A(BC), A(B + C), AB + AC and determine which expressions are equivalent when given 2 × 2 matrices A, B, and C. N.VM.I.9.BA

BeB. (+) Does not meet the basic performance level. N.VM.I.9.BEB

10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. N.VM.I.10

Ad. (+) In addition to Proficient, the Advanced student is able to calculate the determinant,|A|, of a square matrix. Find the inverse, A⁻¹, using the determinant, |A|. Show that A(A⁻¹ is equal to the identity matrix (I). N.VM.I.10.AD

P. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. N.VM.I.10.P

Ba. (+)Calculate AI, IA, A + 0, 0 + A, and determine which expressions are equivalent to A when given 2×2 matrices A, the zero matrix (0), and the identity matrix (I). N.VM.I.10.BA

BeB. (+) Does not meet the basic performance level. N.VM.I.10.BEB

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. N.VM.I.11

Ad. (+) Create a polygon on a coordinate grid, develop the appropriate matrix to represent the polygon, use vectors to translate the shape, and graph the translated polygon on the same coordinate grid. N.VM.I.11.AD

P. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. N.VM.I.11.P

Ba. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. N.VM.I.11.BA

BeB. (+) Does not meet the basic performance level. N.VM.I.11.BEB

12. (+) Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. N.VM.I.12

Ad. (+) Create a polygon on a coordinate grid, develop the appropriate matrix to represent the polygon, use vectors to transform the shape, graph the transformed polygon on the same coordinate grid, find the area for each polygon, and compare the areas. N.VM.I.12.AD

P. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. N.VM.I.12.P

Ba. (+) Identify the transformation that is created by a given 2×2 transformation matrix. N.VM.I.12.BA

BeB. (+) Does not meet the basic performance level. N.VM.I.12.BEB

Seeing Structure In Expressions

A. **Interpret the structure of expressions.** A.SSE.A

1. Interpret expressions that represent a quantity in terms of its context. A.SSE.A.1

A. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.A.1.A

B. Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.A.1.B

Ad. The Advanced student is able to:
Interpret the effect of changes made to a term, a factor, or a coefficient in an expression. OR
Compose complicated expressions from simpler ones and decompose complicated expressions into simpler ones. A.SSE.A.1.AD

P. Interpret expressions that represent a quantity in terms of its context. A.SSE.A.1.P

A. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.A.1.P.A

B. Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.A.1.P.B

Ba. Interpret expressions that represent a quantity in terms of its context by interpreting parts of an expression, such as terms, factors, and coefficients. A.SSE.A.1.BA

BeB. Interpret expressions that represent a quantity in terms of its context by interpreting parts of an expression, such as terms, factors, or coefficients. A.SSE.A.1.BEB

2. Use the structure of an expression to identify ways to rewrite it. A.SSE.A.2

Ad. Explain why various forms of equivalent expressions are more advantageous in a given situation. A.SSE.A.2.AD

P. Use the structure of an expression to identify ways to rewrite it. A.SSE.A.2.P

Ba. Rewrite an expression into another equivalent form. A.SSE.A.2.BA

BeB. May be able to determine if two given expressions are equivalent. A.SSE.A.2.BEB

B. Write expressions in equivalent forms to solve problems. A.SSE.B

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.B.3

A. Factor a quadratic expression to reveal the zeros of the function it defines. A.SSE.B.3.A

B. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. A.SSE.B.3.B

C. Use the properties of exponents to transform expressions for exponential functions. Apply the concepts of decimal and scientific notation to solve real-world and mathematical problems. A.SSE.B.3.C

i. Multiply and divide numbers expressed in both decimal and scientific notation. A.SSE.B.3.C.I

ii. Add and subtract numbers in scientific notation with the same integer exponent. A.SSE.B.3.C.II

Ad. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.B.3.AD

A. Factor a quadratic expression to reveal the zeros of the function it defines and interpret the results in a real-world context. A.SSE.B.3.AD.A

B. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines and interpret the results in a real-world context. A.SSE.B.3.AD.B

P. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.B.3.P

A. Factor a quadratic expression to reveal the zeros of the function it defines A.SSE.B.3.P.A

B. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. A.SSE.B.3.P.B

C. Use the properties of exponents to transform expressions for exponential functions. Apply the concepts of decimal and scientific notation to solve real-world and mathematical problems. A.SSE.B.3.P.C

i. Multiply and divide numbers expressed in both decimal and scientific notation. A.SSE.B.3.P.C.I

ii. Add and subtract numbers in scientific notation with the same integer exponent. A.SSE.B.3.P.C.II

Ba. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
Factor a quadratic expression to reveal the zeros of the function it defines. AND
Use the properties of exponents to transform expressions for exponential functions. A.SSE.B.3.BA

BeB. May be able to choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
Factor a quadratic expression to reveal the zeros of the function it defines. OR
Use the properties of exponents to transform expressions for exponential functions. A.SSE.B.3.BEB

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. A.SSE.B.4

Ad. The Advanced student is able to:
Use the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve real-world problems. OR
Write the series in proper summation notation. A.SSE.B.4.AD

P. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. A.SSE.B.4.P

Ba. Use the given formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. A.SSE.B.4.BA

BeB. May be able to:
Write a geometric sequence as a finite geometric series and calculate its sum. AND
Identify the common ratio and initial term of a finite geometric series (when the common ratio is not 1). A.SSE.B.4.BEB

Arithmetic With Polynomials And Rational Expressions

C. **Perform arithmetic operations on polynomials.** A.APR.C

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.APR.C.1

Ad. The Advanced student is able to:
Rewrite a polynomial expression involving multiplication, and addition or subtraction, into an equivalent polynomial expression in standard form. OR
Generalize a pattern when adding, subtracting, and multiplying polynomials of a varying number of terms. A.APR.C.1.AD

P. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.APR.C.1.P

Ba. Add, subtract, and multiply polynomials. A.APR.C.1.BA

BeB. May be able to add, subtract, and multiply binomials. A.APR.C.1.BEB

D. Understand the relationship between zeros and factors of polynomial. A.APR.D

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by (a - x) is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x). A.APR.D.2

Ad. Find a missing coefficient m, where m is a real number, of a polynomial p(x), when given (x - a) is a factor of p(x) or when given p(a). A.APR.D.2.AD

P. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by (x - a) is p(a) = 0 if and only if (x - a) is a factor of p(x). A.APR.D.2.P

Ba. Evaluate p(a) and compare it to the remainder of p(x)/(x - a). Explain the significance of the remainder. A.APR.D.2.BA

BeB. May be able to evaluate p(a) and compare it to the remainder of p(x)/(x - a). Explain the significance of the remainder. A.APR.D.2.BEB

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A.APR.D.3

Ad. Given a graph of a polynomial function with integer x-intercepts, write the general form of the polynomial in standard form, understanding that the polynomial could have a stretch or compression. A.APR.D.3.AD

P. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A.APR.D.3.P

Ba. Identify zeros of polynomials when suitable factorizations are available, and locate the zeros on a coordinate plane. A.APR.D.3.BA

BeB. May be able to match the polynomial to its factored form and its graph. A.APR.D.3.BEB

E. Use polynomial identities to solve problems. A.APR.E

4. Prove polynomial identities and use them to describe numerical relationships. A.APR.E.4

Ad. Use the structure of a polynomial identity to give real-world contextual meaning to the identity (e.g., completing the square of a quadratic function to highlight the maximum or minimum, factoring a polynomial to highlight the zeros, or factoring a trinomial to highlight base times width). A.APR.E.4.AD

P. Prove polynomial identities and use them to describe numerical relationships. A.APR.E.4.P

Ba. Evaluate each polynomial expression for given values, compare the results, and make a general statement concerning the polynomials as identities. A.APR.E.4.BA

BeB. May be able to given two polynomial identities, evaluate the polynomials at a given value to demonstrate that the polynomials are identities. A.APR.E.4.BEB

5. (+) Know and apply the Binomial Theorem for the expansion of (x + y) ⁿ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. A.APR.E.5

Ad. (+) Find specified terms in the expansion of (x + y)ⁿ by applying properties of the binomial theorem. A.APR.E.5.AD

P. (+) Know and apply the Binomial Theorem for the expansion of (x + y) ⁿ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. A.APR.E.5.P

Ba. (+) Expand (x + y)ⁿ in powers of x and y for a positive integer n, where x and y are any numbers, using Pascal's Triangle. A.APR.E.5.BA

BeB. (+) Does not meet the basic performance level. A.APR.E.5.BEB

F. Rewrite rational expressions. A.APR.F

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x) using inspection, long division, or, for the more complicated examples, a computer algebra system. (i.e. rewriting a rational expression as the quotient plus the remainder over divisor). A.APR.F.6

Ad. Given a(x)/b(x) = q(x) + r(x)/b(x):
Identify the missing coefficient from a polynomial a(x) when given b(x), q(x), and r(x). OR
Generalize the patterns obtained by dividing various degrees of polynomials. A.APR.F.6.AD

P. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x) using inspection, long division, or, for the more complicated examples, a computer algebra system. (i.e. rewriting a rational expression as the quotient plus the remainder over divisor). A.APR.F.6.P

Ba. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x) using inspection or long division where a(x) is a quadratic and b(x) is linear. A.APR.F.6.BA

BeB. May be able to:
Match a rational expression in the form a(x)/b(x) to its equivalent form q(x) + r(x)/b(x). OR
Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x) using inspection or long division where a(x) is a quadratic and b(x) is linear with assistance. A.APR.F.6.BEB

7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. A.APR.F.7

Ad. (+) Justify that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. A.APR.F.7.AD

P. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. A.APR.F.7.P

Ba. (+) Add, subtract, multiply, and divide rational expressions. A.APR.F.7.BA

BeB. (+) Does not meet the basic performance level. A.APR.F.7.BEB

Creating Equations

G. Create equations that describe numbers or relationships. A.CED.G

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.G.1

Ad. Create equations and inequalities in one variable and use them to solve problems. Include compound inequalities arising from problems. Use interval notation to represent inequalities. A.CED.G.1.AD

P. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.G.1.P

Ba. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and simple exponential functions. A.CED.G.1.BA

BeB. May be able to create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions. A.CED.G.1.BEB

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.G.2

Ad. The Advanced student is able to:
Interpret the relationships between the graph and its corresponding equation in real-world contexts. OR
Graph equations of the form x + y + z = c or ordered triples on the xyz-plane. OR
Graph equations from a real-world context of the form y = ab^x where a is a real number and b is greater than 0. A.CED.G.2.AD

P. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.G.2.P

Ba. The Basic student is able to:
Create equations in two or more variables to represent relationships between quantities. OR
Graph equations on coordinate axes with labels and scales. A.CED.G.2.BA

BeB. May be able to match a graph to its equation in two or more variables. A.CED.G.2.BEB

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. A.CED.G.3

Ad. Examine and explain constraints and solutions to systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. A.CED.G.3.AD

P. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. A.CED.G.3.P

Ba. The Basic student is able to:
Interpret solutions as viable or non-viable options in a modeling context. AND
Represent constraints by equations or inequalities, or by systems of equations and/or inequalities. A.CED.G.3.BA

BeB. May be able to:
Interpret solutions as viable or non-viable options in a modeling context. AND
Represent constraints by equations or inequalities. A.CED.G.3.BEB

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.CED.G.4

Ad. Choose and explain reasoning for highlighting the quantity of interest, rearrange the formula, use the rearranged formula to evaluate, and interpret the answer in a real-world context. A.CED.G.4.AD

P. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.CED.G.4.P

Ba. Rearrange formulas to highlight a quantity of interest in two steps, using the same reasoning as in solving equations. A.CED.G.4.BA

BeB. May be able to rearrange formulas to highlight a quantity of interest in one step, using the same reasoning as in solving equations. A.CED.G.4.BEB

Reasoning With Equations And Inequalities

H. Understand solving equations as a process of reasoning and explain the reasoning. A.REI.H

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.H.1

Ad. Critique the solution and justification of self and others in the steps in solving linear equations. A.REI.H.1.AD

P. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.H.1.P

Ba. Informally describe the steps in solving multi-step linear equations. A.REI.H.1.BA

BeB. May be able to match steps to justifications when provided with the steps for solving multi-step linear equations. A.REI.H.1.BEB

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. A.REI.H.2

Ad. Critique the solutions of self and others for simple rational and radical equations in one variable. A.REI.H.2.AD

P. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. A.REI.H.2.P

Ba. Solve simple rational and radical equations in one variable with no extraneous solutions, and when given equations with extraneous solution(s) demonstrate why the given solution(s) is/are not viable. A.REI.H.2.BA

BeB. May be able to solve simple rational and radical equations in one variable with no extraneous solutions. A.REI.H.2.BEB

I. Solve equations and inequalities in one variable. A.REI.I

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.I.3

Ad. The Advanced student is able to:
Critique the solutions of self and others for linear inequalities in one variable, including equations with coefficients represented by letters. OR
Demonstrate the solution of a linear equation or inequality in multiple ways (e.g., graphically, set notation, interval notation). OR
Create and solve a real-world linear equation or inequality in one variable, determine its viable domain and solution set. A.REI.I.3.AD

P. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.I.3.P

Ba. Solve linear equations and inequalities in one variable, limited to numerical coefficients. A.REI.I.3.BA

BeB. May be able to solve linear equations and inequalities in one variable, limited to numerical coefficients, where the variable is on only one side of the equal or inequality sign. A.REI.I.3.BEB

4. Solve quadratic equations in one variable. A.REI.I.4

   A. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. A.REI.I.4.A

   B. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. A.REI.I.4.B

   C. (+) Derive the quadratic formula from the general form of a quadratic equation. A.REI.I.4.C

Ad. The Advanced student is able to:
Critique different methods used by self and others for solving quadratic equations in one variable. OR
Create and solve a real-world quadratic equation in one variable, determine its viable domain and solution. OR
Explain the purpose for the method chosen to solve the quadratic equation in a real-world situation. A.REI.I.4.AD

P. Solve quadratic equations in one variable. A.REI.I.4.P

   A. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. A.REI.I.4.P.A

   B. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. A.REI.I.4.P.B

   C. (+) Derive the quadratic formula from the general form of a quadratic equation. A.REI.I.4.P.C

Ba. Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$), taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. A.REI.I.4.BA

BeB. May be able to solve quadratic equations in one variable, using the quadratic formula. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. A.REI.I.4.BEB

J. Solve systems of equations. A.REI.J

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other, produces a system with the same solutions. A.REI.J.5

Ad. In addition to Proficient, the Advanced student is able to:
Write a different system of two equations in two variables with the same solution as a given system of two equations in two variables. OR
Create and solve, if possible, a system of two equations in two variables for a real-world context (include systems with one solution, infinitely many solutions, or no solution). A.REI.J.5.AD

P. Prove that given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other, produces a system with the same solutions. A.REI.J.5.P

Ba. Solve a system of two equations in two variables with different coefficients resulting in one solution using the elimination method. A.REI.J.5.BA

BeB. Solve a system of two equations in two variables with equal or opposite coefficients resulting in one solution using the elimination method. A.REI.J.5.BEB

6. Estimate solutions graphically and determine algebraic solutions to linear systems, focusing on pairs of linear equations in two variables. A.REI.J.6

Ad. Approximate solutions to a system of linear equations for a real-world situation using a table and graph, then verify the solution algebraically and discuss the viability of the solution in the context of the problem. A.REI.J.6.AD

P. Estimate solutions graphically and determine algebraic solutions to linear systems, focusing on pairs of linear equations in two variables. A.REI.J.6.P

Ba. Estimate solutions to linear systems graphically or determine algebraic solutions to linear systems, focusing on pairs of linear equations in two variables. A.REI.J.6.BA

BeB. May be able to test a solution to the system in both original equations (graphically or algebraically). A.REI.J.6.BEB

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A.REI.J.7

Ad. Explore algebraically, graphically, and tabularly the number and type of the solutions of one linear and one quadratic or two quadratics and describe findings. A.REI.J.7.AD

P. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A.REI.J.7.P

Ba. Solve a simple system consisting of a linear equation written in slope-intercept form and a quadratic equation written in standard form in two variables algebraically. A.REI.J.7.BA

BeB. May be able to solve a simple system consisting of a linear equation written in slope-intercept form and a quadratic equation written in standard form in two variables graphically. A.REI.J.7.BEB

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. A.REI.J.8

Ad. (+) Represent a real-world situation using a system of linear equations as a single matrix equation in a vector variable. A.REI.J.8.AD

P. (+) Represent a system of linear equations as a single matrix equation in a vector variable. A.REI.J.8.P

Ba. (+) Identify a system of linear equations given a matrix equation in a vector variable. A.REI.J.8.BA

BeB. (+) Does not meet the basic performance level. A.REI.J.8.BEB

9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). A.REI.J.9

Ad. (+) Find the inverse of a matrix if it exists and use it to solve a real-world problem involving systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). A.REI.J.9.AD

P. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). A.REI.J.9.P

Ba. (+) Solve systems of linear equations written as a single matrix equation in a vector variable when given the inverse of a matrix (using technology for matrices of dimension 3 × 3 or greater). A.REI.J.9.BA

BeB. (+) Does not meet the basic performance level. A.REI.J.9.BEB

K. Represent and solve equations and inequalities graphically. A.REI.K

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. A.REI.K.10

Ad. Create and graph two equations of different degrees that pass through the same two specific points. Describe the relationship between the solution sets for each equation and the solution for the system. A.REI.K.10.AD

P. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane (i.e., connect algebraic and graphical representations of an equation in two variables). A.REI.K.10.P

Ba. Create and verify a set of ordered pairs that lie on the graph when given an equation. A.REI.K.10.BA

BeB. May be able to determine which ordered pairs do or do not lie on the graph of the equation when given an equation and a set of ordered pairs. A.REI.K.10.BEB

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A.REI.K.11

Ad. Given a table of ordered pairs and graph for both a linear y = f(x) and quadratic y = g(x) where the intersection is a non-integer coordinate pair, describe a method to find the equations and solutions matching the depicted graphs and tables. Find the solutions using the generated equations. Describe the accuracy of the solution algebraically and graphically. Describe how to improve the method. A.REI.K.11.AD

P. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A.REI.K.11.P

Ba. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions or make tables of values. Include cases where f(x) and/or g(x) are linear, quadratic, absolute value, and exponential. A.REI.K.11.BA

BeB. May be able to explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions or make tables of values. Include cases where f(x) and/or g(x) are linear, quadratic, and exponential. A.REI.K.11.BEB

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. A.REI.K.12

Ad. The Advanced student is able to:
Write a system of linear inequalities that includes a set of specified points in a region and explain the choice of strict inequality or non-strict inequality notation. OR
Create a system of linear inequalities that will optimize the solution when given a real-life scenario. OR
Write the inequalities that describe the boundaries of that scenario and discuss the optimal solution when given a graphical representation of a real-life scenario. A.REI.K.12.AD

P. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. A.REI.K.12.P

Ba. Graph the solution to a linear inequality in two variables as a half-plane (excluding the boundary in the case of strict inequality). A.REI.K.12.BA

BeB. The Below Basic student may be able to match the solution to a linear inequality in two variables to its graph half-plane (excluding the boundary in the case of strict inequality). A.REI.K.12.BEB

Interpreting Functions

A. Understand the concept of a function and use function notation. F.IF.A

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.A.1

Ad. Use multiple representations to generalize ways to define functions and non-functions. F.IF.A.1.AD

P. Demonstrate that a function's domain is assigned to exactly one element of the range in equations, tables, graphs, and context. F.IF.A.1.P

Ba. Demonstrate that a function's domain is assigned to exactly one element of the range in equations, tables, and graphs. F.IF.A.1.BA

BeB. May be able to demonstrate that a function's domain is assigned to exactly one element of the range in tables and graphs. F.IF.A.1.BEB

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.A.2

Ad. Create context from a given domain and range and use function notation to write an equation, graph the function, and draw a picture that models the context. F.IF.A.2.AD

P. Use function notation, evaluate functions for inputs in their domain, and interpret statements that use function notation in terms of a context. F.IF.A.2.P

Ba. Use function notation and evaluate functions for inputs in their domain. F.IF.A.2.BA

BeB. May be able to evaluate equations for specific values and match the equation to function notation. F.IF.A.2.BEB

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.IF.A.3

Ad. Write the explicit and recursive forms of a sequence describing linear and exponential situations. Express the sequence graphically and in a table. F.IF.A.3.AD

P. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.IF.A.3.P

Ba. Write an informal recursive description of a sequence. F.IF.A.3.BA

BeB. May be able to write the first n terms when given an informal recursive description of a sequence. F.IF.A.3.BEB

B. Interpret functions that arise in applications in terms of the context. F.IF.B

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.B.4

Ad. Graph and label the key features of the function and write the function in function notation when given key features from a linear, exponential, or quadratic context. F.IF.B.4.AD

P. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. F.IF.B.4.P

Ba. The Basic student is able to, for linear, quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, maximums and minimums; symmetries; end behavior. F.IF.B.4.BA

BeB. The Below Basic student may be able to, for linear, quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs in terms of the quantities, and sketch graphs showing key features given the function. Key features include: intercepts; intervals where the function is increasing, decreasing, maximums and minimums; symmetries; end behavior. F.IF.B.4.BEB

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.B.5

Ad. In addition to Proficient, the Advanced student is able to:
Write and graph a function for a given context where the domain meets given parameters. Express the domain of the function using interval and/or set notation as appropriate. OR
Given the graph of a function, write its domain and range using interval and/or set notation as appropriate. F.IF.B.5.AD

P. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.B.5.P

Ba. Match the domain of a function to its graph and explain why the selected domain is applicable to the situation. F.IF.B.5.BA

BeB. May be able to match the domain of a function to its graph. F.IF.B.5.BEB

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.IF.B.6

Ad. The Advanced student is able to:
Analyze the difference between the rates of change of different types of functions. OR
Compare average rates of change over different intervals of the same function. OR
Generalize how the average rate of change differs between different function types. F.IF.B.6.AD

P. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.IF.B.6.P

Ba. Calculate and interpret the average rate of change of a linear, exponential, or quadratic function over a specified interval presented as a graph, an equation, or a table. F.IF.B.6.BA

BeB. May be able to calculate and interpret the average rate of change of a linear, exponential, or quadratic function over a specified interval presented as a graph. F.IF.B.6.BEB

C. Analyze functions using different representations. F.IF.C

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.C.7

   A. Graph linear and quadratic functions and show intercepts, maxima, and minima. F.IF.C.7.A

   B. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.C.7.B

   C. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.IF.C.7.C

   D. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. F.IF.C.7.D

   E. Graph exponential and logarithmic functions, showing intercepts and end behavior. F.IF.C.7.E

   F. (+) Graph trigonometric functions, showing period, midline, and amplitude. F.IF.C.7.F

Ad. Create different representations of linear, quadratic, and exponential functions when given one of the following representations: graphical, tabular, or algebraic. Compare and contrast all three function types identifying key features while referencing the representations. F.IF.C.7.AD

P. Graph linear, quadratic, and exponential functions expressed symbolically and show appropriate key features of the graph showing intercepts, maxima, and minima, and end behavior. F.IF.C.7.P

   A. Graph linear and quadratic functions and show intercepts, maxima, and minima. F.IF.C.7.P.A

   B. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.C.7.P.B

   C. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.IF.C.7.P.C

   E. Graph exponential and logarithmic functions, showing intercepts and end behavior F.IF.C.7.P.E

   D. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. F.IF.C.7.P.D

   F. (+) Graph trigonometric functions, showing period, midline, and amplitude. F.IF.C.7.P.F

Ba. Identify appropriate key features of linear, quadratic, and exponential functions from a graph showing intercepts (linear, quadratic, and exponential), maximum or minimum (quadratic), and end behavior (linear, quadratic, and end behavior). F.IF.C.7.BA

A. Graph linear and quadratic functions and show intercepts (linear and quadratic) and maxima or minima (quadratic). F.IF.C.7.BA.A

B. Graph square root, cube root, and absolute value functions. F.IF.C.7.BA.B

C. Graph polynomial functions, identifying zeros and showing end behavior. F.IF.C.7.BA.C

E. Graph exponential and logarithmic functions, showing intercepts and end behavior. F.IF.C.7.BA.E

D. (+) Graph rational functions, identifying zeros and asymptotes, and showing end behavior. F.IF.C.7.BA.D

F. (+) Graph trigonometric functions, showing period, midline, and amplitude. F.IF.C.7.BA.F

BeB. Match descriptions of key features of linear, quadratic, and exponential functions to the appropriate parts of the graph including intercepts (linear, quadratic, and exponential), maximum or minimum (quadratic), and end behavior (linear, quadratic, and end behavior). F.IF.C.7.BEB

A. For linear and quadratic functions, identify intercepts (linear and quadratic) and maxima or minima (quadratic). F.IF.C.7.BEB.A

B. For square root, cube root, and absolute value functions identify intercepts, symmetry, and end behavior. F.IF.C.7.BEB.B

C. For polynomial functions, identify intercepts and end behavior. F.IF.C.7.BEB.C

E. For exponential and logarithmic functions, identify intercepts and end behavior. F.IF.C.7.BEB.E

D. (+) For rational functions, identify zeros, asymptotes, and end behavior. F.IF.C.7.BEB.D

F. (+) For trigonometric functions with no vertical shift, identify amplitude and period using a set of x-intercepts. F.IF.C.7.BEB.F

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.C.8

A. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F.IF.C.8.A

B. Use the properties of exponents to interpret expressions for exponential functions. F.IF.C.8.B

Ad. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.C.8.AD

A. Compare and contrast factored, vertex, and standard form to discuss the appropriate form of various properties (zeros, extreme values, and symmetry). Write a quadratic function in different forms to reveal the appropriate properties and compare those properties to the function's graph or table of values in a real-world context. F.IF.C.8.AD.A

B. Write an exponential function from a real-world context and use the properties of exponents to interpret the expression in the context of the situation. F.IF.C.8.AD.B

P. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.C.8.P

A. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F.IF.C.8.P.A

B. Use the properties of exponents to interpret expressions for exponential functions. F.IF.C.8.P.B

Ba. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.C.8.BA

A. Use the process of factoring to show zeros. Compare standard and vertex forms of a quadratic function to show extreme values and symmetry of the graph. Interpret these in terms of a context. F.IF.C.8.BA.A

B. Use the properties of exponents to evaluate expressions for exponential functions. F.IF.C.8.BA.B

BeB. May be able to write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.C.8.BEB

A. Match the factored form to its graph to identify zeros. Match vertex form of a quadratic function to its graph to show extreme values and symmetry of the graph. F.IF.C.8.BEB.A

B. Match properties of exponential functions to the appropriate part of the exponential expression. F.IF.C.8.BEB.B

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.C.9

Ad. Create different representations of two functions (algebraically, graphically, numerically in tables, or by verbal description) from a given representation. Compare and contrast properties of those functions, specifically highlighting what each representation reveals. F.IF.C.9.AD

P. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.C.9.P

Ba. Compare properties of two functions when given only two different representations at a time (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.C.9.BA

BeB. May be able to match properties of two functions when given only two different representations at a time (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.C.9.BEB

Building Functions

D. Build a function that models a relationship between two quantities. F.BF.D

1. Write a function that describes a relationship between two quantities. F.BF.D.1

   A. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.D.1.A

   B. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.BF.D.1.B

   C. (+) Compose functions. F.BF.D.1.C

   Ad. Write a function that describes a relationship between two quantities. F.BF.D.1.AD

   A. Write two or more explicit expressions to express a single sequence shown pictorially and compare their features. F.BF.D.1.AD.A

   B. Graphically and tabularly combine standard function types using arithmetic operations. F.BF.D.1.AD.B

   C. (+) Determine which field properties hold under compositions. Determine under what conditions the Commutative Property holds for composition. F.BF.D.1.AD.C

   P. Write a function that describes a relationship between two quantities. F.BF.D.1.P

   A. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.D.1.P.A

   B. Combine standard function types using arithmetic operations. F.BF.D.1.P.B

   C. (+) Compose functions. F.BF.D.1.P.C

   Ba. Write a function that describes a relationship between two quantities. F.BF.D.1.BA

   A. Determine an explicit expression, a recursive process, or steps for calculation from a context for linear and exponential relationships. F.BF.D.1.BA.A

   B. Combine standard function types using arithmetic operations for addition, subtraction, and multiplication of binomials. F.BF.D.1.BA.B

   C. (+) Compose functions numerically and graphically, and interpret the solution in context. F.BF.D.1.BA.C

BeB. May be able to write a function that describes a relationship between two quantities. F.BF.D.1.BEB

A. Determine an explicit expression, a recursive process, or steps for calculation from a context for linear relationships. F.BF.D.1.BEB.A

B. Combine standard function types using arithmetic operations for addition, subtraction, and multiplication of linear binomials. F.BF.D.1.BEB.B

C. (+) Compose functions numerically, and interpret the solution in context. F.BF.D.1.BEB.C

2. (+) Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.BF.D.2

Ad. (+) The Advanced student is able to:
Compare the graphical representation, tabular representation, explicit and recursive formulas, and contextual representation for arithmetic and geometric sequences. Draw parallels to linear and exponential functions, respectively. AND/OR Describe a real-world situation that can be modeled linearly or exponentially and develop the explicit or recursive formulas to model the situation. F.BF.D.2.AD

P. (+) Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.BF.D.2.P

Ba. (+) Write arithmetic and geometric sequences both recursively and with an explicit formula given a modeling situation. F.BF.D.2.BA

BeB. (+) May be able to write an explicit or recursive formula for the model of an arithmetic and geometric sequence given the other formula. F.BF.D.2.BEB

E. Build new functions from existing functions. F.BF.E

3. Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.E.3

Ad. The Advanced student is able to:
Write the equation for a transformed parent function given the graph or verbal description of the transformations. OR
Write a description of the transformations using function notation given a verbal description of the transformations of f(x) or the original f(x) graph and its transformed graph. OR
Generalize effects of a transformation on domains and ranges, including effects on ordered pairs, when exploring all of the different types of transformations in multiple representations of f(x). F.BF.E.3.AD

P. Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.E.3.P

Ba. Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. F.BF.E.3.BA

BeB. May be able to match the equation to the graph showing the effect on the graph of replacing f(x) by f(x) + k, kf(x), and f(x + k) for specific values of k (both positive and negative); write a description of the transformation. Experiment with cases and illustrate an explanation of the effects on the graph using technology. F.BF.E.3.BEB

4. Find inverse functions. F.BF.E.4

   A. Write an expression for the inverse of a simple, invertible function f(x). Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, as f and g are inverse functions, if and only if, f(x) = y and g(y) = x, for all values of x in the domain of f and all values of y in the domain of g. F.BF.E.4.A

   B. (+) Verify by composition that one function is the inverse of another. F.BF.E.4.B

   C. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.E.4.C

   D. (+) Produce an invertible function from a non-invertible function by restricting the domain. F.BF.E.4.D

Ad. Find inverse functions. F.BF.E.4.AD

   A. Create a model of a function and its inverse from a real-world context. F.BF.E.4.AD.A

   B. (+) Compare the equation, table, and graph when composing two functions f(g(x)) and g(f(x)). Determine what happens to the equations, table values, and graphs of f(g(x)) and g(f(x)) when f and g are inverses. F.BF.E.4.AD.B

   C. (+) Create a table for two functions in such a way that one is invertible and the other is not. Create graphs for the functions and connect features of the graph to properties of invertible functions. F.BF.E.4.AD.C

   D. (+) Show the relationship between the properties of a function and its inverse (domain, range, increasing, decreasing, asymptotes, intercepts). F.BF.E.4.AD.D

P. Find inverse functions. F.BF.E.4.P

A. Write an expression for the inverse of a simple, invertible function f(x). Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, as f and g are inverse functions, if and only if, f(x) = y and g(y) = x, for all values of x in the domain of f and all values of y in the domain of g. F.BF.E.4.P.A

B. (+) Verify by composition that one function is the inverse of another. F.BF.E.4.P.B

C. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.E.4.P.C

D. (+) Produce an invertible function from a non-invertible function by restricting the domain. F.BF.E.4.P.D

Ba. Understand inverse functions. F.BF.E.4.BA

A. Match expressions for a function and its inverse. F.BF.E.4.BA.A

B. (+) Identify the compositions that shows f and g are inverses. F.BF.E.4.BA.B

C. (+) Identify values of an inverse function from a table, given that the function has an inverse. F.BF.E.4.BA.C

D. (+) Restrict the domain of a non-invertible function to make it invertible given a graph. F.BF.E.4.BA.D

BeB. May be able to understand inverse functions. F.BF.E.4.BEB

A. Match the graph and/or table of a function to the graph and/or table of its inverse. F.BF.E.4.BEB.A

B. (+) Show that one function may be the inverse of another by using numerical composition. F.BF.E.4.BEB.B

C. (+) Create a table for an inverse of a function, given a function table. F.BF.E.4.BEB.C

D. (+) Determine if a function will have an inverse from a given graph. F.BF.E.4.BEB.D

5. (+) Build new functions from existing functions. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F.BF.E.5

Ad. (+) Solve real-world problems involving logarithms and exponents. F.BF.E.5.AD

P. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F.BF.E.5.P

Ba. (+) The Basic student is able to:
Given an expression written in logarithmic form, write the equivalent expression in exponential form. AND
Given an expression in exponential form, write the equivalent expression in logarithmic form. F.BF.E.5.BA

BeB. (+) Does not meet the basic performance level. F.BF.E.5.BEB

Linear, Quadratic, And Exponential Models

F. Construct and compare linear, quadratic, and exponential models and solve problems. F.LE.F

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.F.1

   A. Verify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. F.LE.F.1.A

   B. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.F.1.B

   C. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.F.1.C

Ad. In addition to Proficient, the Advanced student is able to distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.F.1.AD

   A. Write an equation, sketch a graph, and create a table of values given linear and exponential real-world contexts and explain how the growth changes. F.LE.F.1.AD.A

   B. Predict and compare values along a continuum of linear and exponential functions in real-world contexts. F.LE.F.1.AD.B

   C. Compare and contrast the rates of change of linear and exponential functions. Write a generalization about rates of change. F.LE.F.1.AD.C

P. Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.F.1.P

   A. Verify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. F.LE.F.1.P.A

   B. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.F.1.P.B

   C. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.F.1.P.C

Ba. Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.F.1.BA

   A. Demonstrate that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals from a table of values. F.LE.F.1.BA.A

   B. Match situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.F.1.BA.B

   C. Match situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.F.1.BA.C

BeB. May be able to distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.F.1.BEB

A. Informally show that linear functions grow by equal differences over equal intervals by calculating rate of change for two sets of order pairs. Informally show that exponential functions grow by equal factors over equal intervals by showing multiplicative growth for two sets of ordered pairs. F.LE.F.1.BEB.A

B. Identify graphs in which one quantity changes at a constant rate per unit interval relative to another. F.LE.F.1.BEB.B

C. Identify graphs in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.F.1.BEB.C

2. Construct linear and exponential functions using a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.F.2

Ad. The Advanced student is able to:
Explain how exponential and linear function values differ as x approaches positive or negative infinity. AND Relate algebraic representations of linear and exponential functions to the explicit and recursive forms of arithmetic and geometric sequences, respectively. AND/OR
Create a real-world scenario and develop linear or exponential tables or graphs representing the scenario. F.LE.F.2.AD

P. Construct linear and exponential functions using a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.F.2.P

Ba. Construct linear and exponential functions using two of the following representations: a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.F.2.BA

BeB. May be able to match linear and exponential functions to their graph, description of a relationship, and two input-output pairs (include reading these from a table). F.LE.F.2.BEB

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. F.LE.F.3

Ad. Use an algebraic argument or informal proof to show that an increasing exponential function eventually exceeds an increasing linear function. F.LE.F.3.AD

P. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. F.LE.F.3.P

Ba. Identify which function eventually exceeds the others when given the graphs and tables of increasing linear, quadratic, and exponential functions. F.LE.F.3.BA

BeB. Compare the output values for increasing x -values of increasing linear, quadratic, and exponential functions to determine which function has the greatest output value. F.LE.F.3.BEB

4. For exponential models, express as a logarithm the solution to ab^{ct} = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. F.LE.F.4

Ad. Construct an exponential model using given contextual data. Predict the input from a graph or table then use logarithms to find the exact solution of an independent value from the context, given a dependent value. Compare and discuss the exact solution to a graphical or tabular solution. F.LE.F.4.AD

P. For exponential models, express as a logarithm the solution to ab^{ct} = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. F.LE.F.4.P

Ba. For exponential models, express as a logarithm the solution to ab ^t = d where a and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. F.LE.F.4.BA

BeB. For exponential models, express as a logarithm the solution to b^twhere d is a real number and the base b is 2, 10, or e; evaluate the logarithm using technology. F.LE.F.4.BEB

G. Interpret expressions for functions in terms of the situation they model. F.LE.G

5. Interpret the parameters in a linear or exponential function in terms of a context. F.LE.G.5

Ad. Model real-world scenarios with linear and exponential functions with appropriate parameters. F.LE.G.5.AD

P. Interpret the parameters in a linear or exponential function in terms of a context. F.LE.G.5.P

Ba. Identify appropriate parameters for linear or exponential functions in terms of a context. F.LE.G.5.BA

BeB. May be able to match parameters for linear or exponential functions to the appropriate parts of the graph. F.LE.G.5.BEB

Trigonometric Functions

H. Extend the domain of trigonometric functions using the unit circle. F.TF.H

1. (+) Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.H.1

Ad. (+) Solve problems using radian measure. F.TF.H.1.AD

P. (+) Demonstrate that radian measure of an angle is the length of the arc on the unit circle subtended by the angle. F.TF.H.1.P

Ba. (+) Convert between degree and radian measures. F.TF.H.1.BA

BeB. (+) Does not meet the basic performance level. F.TF.H.1.BEB

2. (+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.H.2

Ad. (+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed clockwise and counterclockwise around the unit circle. F.TF.H.2.AD

P. (+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.H.2.P

Ba. (+) Explain how the unit circle in the coordinate plane enables the extension of the sine and cosine functions to the special angles (e.g., $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$), interpreted as radian measures of angles traversed counterclockwise once around the unit circle. F.TF.H.2.BA

BeB. (+) Does not meet the basic performance level. F.TF.H.2.BEB

3. (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.H.3

Ad. (+) Use special triangles to determine geometrically the values of the six trigonometric functions for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of the six trigonometric functions for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.H.3.AD

P. (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.H.3.P

Ba. (+) Use special triangles to determine geometrically the values of sine and cosine for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine and cosine for the reference angles of $\pi/3$, $\pi/4$, and $\pi/6$. F.TF.H.3.BA

BeB. (+) Does not meet the basic performance level. F.TF.H.3.BEB

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.H.4

Ad. (+) Verify the relationships for symmetry (odd and even) of trigonometric functions holds for values of theta outside of the first quadrant of the unit circle. F.TF.H.4.AD

P. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.H.4.P

Ba. (+) Use the unit circle to explain periodicity of trigonometric functions. F.TF.H.4.BA

BeB. (+) Does not meet the basic performance level. F.TF.H.4.BEB

I. Model periodic phenomena with trigonometric functions. F.TF.I

5. (+) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.I.5

Ad. (+) Model trigonometric functions of periodic phenomena by identifying amplitude, frequency, and midline, and writing the equation when given a data set. F.TF.I.5.AD

P. (+) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.I.5.P

Ba. (+) Choose trigonometric functions to model periodic phenomena with specified amplitude and midline. F.TF.I.5.BA

BeB. (+) Does not meet the basic performance level. F.TF.I.5.BEB

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.I.6

Ad. (+) Identify key features of an inverse trigonometric function when given a trigonometric function whose domain is always increasing or always decreasing. F.TF.I.6.AD

P. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.I.6.P

Ba. (+) Understand that restricting the sine and cosine functions to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.I.6.BA

BeB. (+) Does not meet the basic performance level. F.TF.I.6.BEB

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. F.TF.I.7

Ad. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; extrapolate solutions to the model based on periodicity, and interpret them in terms of the context. F.TF.I.7.AD

P. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. F.TF.I.7.P

Ba. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology. F.TF.I.7.BA

BeB. (+) Does not meet the basic performance level. F.TF.I.7.BEB

J. Prove and apply trigonometric identities. F.TF.J

8. (+) Prove the Pythagorean identity $\sin^2 A + \cos^2 A = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A, and the quadrant of the angle. F.TF.J.8

Ad. (+) Prove the other two Pythagorean identities using $\sin^2 A + \cos^2 A = 1$. F.TF.J.8.AD

P. (+) Prove the Pythagorean identity $\sin^2 A + \cos^2 A = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A and the quadrant of the angle. F.TF.J.8.P

Ba. (+) Use the Pythagorean identity $\sin^2 A + \cos^2 A = 1$ to find sin A or cos A given sin A or cos A and the quadrant of the angle. F.TF.J.8.BA

BeB. (+) Does not meet the basic performance level. F.TF.J.8.BEB

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. F.TF.J.9

Ad. (+) Use the addition formula to prove multiple-angle identities and use them to solve problems. F.TF.J.9.AD

P. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. F.TF.J.9.P

Ba. (+) Use the addition and subtraction formulas for sine and cosine to solve problems. F.TF.J.9.BA

BeB. (+) Does not meet the basic performance level. F.TF.J.9.BEB

Congruence

A. **Experiment with transformations in the plane.** G.CO.A

1. Apply precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.A.1

Ad. Apply and justify the use of precise definitions while synthetically and/or analytically solving problems. G.CO.A.1.AD

P. Apply precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.A.1.P

Ba. Define an angle, circle, perpendicular line, parallel line, and line segment in simple terms. G.CO.A.1.BA

BeB. May be able to identify an angle, circle, perpendicular line, parallel line, and line segment in simple terms given contextual and/or illustrative choices. G.CO.A.1.BEB

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.A.2

Ad. In addition to Proficient, the Advanced student is able to generalize the representations of rigid transformations by using and recognizing transformations in other areas of mathematics. G.CO.A.2.AD

P. The Proficient student is able to represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.A.2.P

Ba. The Basic student is able to perform two of the following:
Represent transformations in the plane using, e.g., transparencies and geometry software. Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.A.2.BA

BeB. The Below Basic student may be able to perform one of the following: Represent transformations in the plane using, e.g., transparencies and geometry software.
Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.A.2.BEB

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.A.3

Ad. In addition to Proficient, the Advanced student is able to utilize reflective and rotational symmetry to describe irregular polygons or algebraic functions. G.CO.A.3.AD

P. The Proficient student is able to, given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.A.3.P

Ba. The Basic student is able to determine if some, but not all, shapes (rectangle, parallelogram, trapezoid, or regular polygon) have rotational and/or reflective symmetry. G.CO.A.3.BA

BeB. The Below Basic student may be able to choose a rectangle, parallelogram, trapezoid, or regular polygon and determine if it has rotational or reflective symmetry and/or identify if a figure has been rotated or reflected when given a graph or picture. G.CO.A.3.BEB

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.A.4

Ad. Apply the definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments to real-world situations. G.CO.A.4.AD

P. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.A.4.P

Ba. Develop definitions of rotations, reflections, and/or translations in terms of angles, circles, perpendicular lines, parallel lines, and/or line segments. G.CO.A.4.BA

BeB. May be able to identify rotations, reflections, and/or translations in terms of angles, circles, perpendicular lines, parallel lines, and/or line segments. G.CO.A.4.BEB

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. G.CO.A.5

Ad. Generate a geometric figure identifying the rotations, reflections, or translations used in its creation. Investigate to determine an efficient sequence of transformations that will carry a given figure onto another. G.CO.A.5.AD

P. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. G.CO.A.5.P

Ba. Sketch the image of the figure when given a geometric figure and a transformation described in words, or given a sketch, describe the transformation in writing or verbally. G.CO.A.5.BA

BeB. May be able to, given a figure, identify and describe the transformation performed on a given shape in simple terms. G.CO.A.5.BEB

B. Understand congruence in terms of rigid motions. G.CO.B

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.B.6

Ad. Use the definition of congruence, in terms of rigid motions, to construct a viable argument that two figures are congruent and/or predict the effect on a given rigid motion on an algebraic function. G.CO.B.6.AD

P. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.B.6.P

Ba. Recognize congruency and/or be able to predict the effect on a rigid motion on a given figure. G.CO.B.6.BA

BeB. May be able to determine if two figures are congruent and explain their reasoning. G.CO.B.6.BEB

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.B.7

Ad. Construct a viable argument using the definition of congruence, in terms of rigid motions, to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent and/or use counter-examples. G.CO.B.7.AD

P. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.B.7.P

Ba. Recognize and identify that two triangles are congruent using rigid transformation. G.CO.B.7.BA

BeB. May be able to distinguish between congruent and non-congruent triangles. G.CO.B.7.BEB

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. G.CO.B.8

Ad. Explain and provide examples showing that the criteria (SSA, AAA, and SAA) do not always prove triangles congruent. G.CO.B.8.AD

P. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. G.CO.B.8.P

Ba. Identify the criteria for triangle congruence (ASA, SAS, and/or SSS) follow from definition of congruence using rigid motions, using tools such as rulers, protractors, distance formula, etc. G.CO.B.8.BA

BeB. May be able to identify corresponding parts in two triangles. G.CO.B.8.BEB

C. Prove geometric theorems. G.CO.C

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. G.CO.C.9

Ad. Prove theorems about lines and angles using multiple representations (constructions, analytic geometry, theorems, etc.) and/or analyze and critique proofs written by others by verifying the logic. G.CO.C.9.AD

P. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. G.CO.C.9.P

Ba. The Basic student is able to:
Give an informal explanation of the relationship of pairs of lines and/or angles. AND/OR
Complete a partial proof by filling in the blanks when given either a statement or a reason. G.CO.C.9.BA

BeB. May be able to identify:
Vertical angles are congruent;
When a transversal crosses parallel lines, alternate interior angles are congruent; AND
Points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. G.CO.C.9.BEB

10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. G.CO.C.10

Ad. The Advanced student is able to:
Prove theorems about triangles using multiple representations (constructions, analytic geometry, theorems, etc.) AND/OR
Analyze and critique proofs written by others by verifying the logic. G.CO.C.10.AD

P. Prove theorems about triangles. Theorems include: measure of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. G.CO.C.10.P

Ba. The Basic student is able to:
Give informal explanations of triangle proofs. AND/OR
Complete a partial proof by filling in the blanks when given either a statement or a reason. Theorems include: measure of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. G.CO.C.10.BA

BeB. May be able to identify triangle theorems. Theorems include: measure of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. G.CO.C.10.BEB

11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. G.CO.C.11

Ad. Using theorems about parallelograms and triangles, prove other conjectures about figures that are compositions of parallelograms and/or triangles and/or circles. G.CO.C.11.AD

P. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. G.CO.C.11.P

Ba. Informally prove, (including by measurement or inspection) theorems about parallelograms, and:
Complete proofs of theorems about parallelograms, AND/OR
Order the steps of a proof of theorems about parallelograms. G.CO.C.11.BA

BeB. May be able to, given a proof and figure:
Identify opposite and consecutive sides or angles in a figure. OR
Identify and define congruent figures using symbols or given notation. OR
Identify and define parallel lines using symbols or given notation. OR
Identify and define perpendicular lines symbols or given notation. OR
Define supplementary angles. OR
Draw or identify diagonals of a polygon. G.CO.C.11.BEB

D. Make geometric constructions. G.CO.D

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. G.CO.D.12

Ad. Use formal geometric constructions to prove theorems about parallelograms, circles, and triangles or prove that the formal geometric construction copies a line segment; copies an angle; bisects a segment; bisects an angle; constructs perpendicular lines, including perpendicular bisectors of a line segment; constructing a line parallel to a given line through a point not on the line. G.CO.D.12.AD

P. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. G.CO.D.12.P

Ba. Make some, but not all, formal geometric constructions using at least one tool or method:
Copying a segment.
Copying an angle.
Bisecting a segment.
Bisecting an angle.
Constructing perpendicular lines, including the perpendicular bisector of a line segment.
Constructing a line parallel to a given line through a point not on the line. G.CO.D.12.BA

BeB. May be able to make some, but not all, formal geometric constructions using at least one tool or method with assistance:
Copying a segment. Copying an angle.
Bisecting a segment.
Bisecting an angle. Constructing perpendicular lines, including the perpendicular bisector of a line segment.
Constructing a line parallel to a given line through a point not on the line. G.CO.D.12.BEB

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. G.CO.D.13

Ad. Construct an equilateral triangle, a square, and other regular polygons (e.g., pentagon, hexagon, octagon) inscribed in a circle and justify the tools and techniques used. G.CO.D.13.AD

P. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. G.CO.D.13.P

Ba. Construct an equilateral triangle, a square, or a regular hexagon inscribed in a circle. G.CO.D.13.BA

BeB. May be able to construct an equilateral triangle, a square, or a regular hexagon inscribed in a circle with assistance (e.g., video, ordering completed steps of a construction, etc.). G.CO.D.13.BEB

Similarity, Right Triangles, And Trigonometry

E. Understand similarity in terms of similarity transformations. G.SRT.E

1. Verify heuristically the properties of dilations given by a center and a scale factor. G.SRT.E.1

   A. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. G.SRT.E.1.A

   B. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.E.1.B

Ad. Prove the properties of dilations given by a center and a scale factor. G.SRT.E.1.AD

A. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. G.SRT.E.1.AD.A

B. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.E.1.AD.B

P. Understand similarity in terms of similarity transformations. Verify heuristically the properties of dilations given by a center and a scale factor. (A heuristic approach is an approach to problem solving or discovery that employs practical method that is not guaranteed to be optimal or perfect, but is sufficient for the immediate goals.) G.SRT.E.1.P

A. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. G.SRT.E.1.P.A

B. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.E.1.P.B

Ba. Demonstrate that two figures are similar using the given center of dilation and the scale factor. G.SRT.E.1.BA

BeB. May be able to recognize that two figures resulting from a dilation are similar. G.SRT.E.1.BEB

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.E.2

Ad. Given two figures, use the definition of similarity in terms of similarity transformations and other theorems to prove that the figures are similar; paying particular attention to the equality of corresponding angle pairs and the proportionality of corresponding side pairs. G.SRT.E.2.AD

P. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.E.2.P

Ba. Given two similar figures, demonstrate that corresponding angle pairs are congruent and that corresponding side pairs are in proportion using transformations and/or measurement. G.SRT.E.2.BA

BeB. May be able to, given two similar figures, identify congruent corresponding angle pairs and corresponding side pairs. G.SRT.E.2.BEB

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. G.SRT.E.3

Ad. Prove that two triangles are similar or dissimilar using the AA criterion for two similar triangles or prove the AA criterion. G.SRT.E.3.AD

P. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. G.SRT.E.3.P

Ba. Given two triangles, determine similarity and dissimilarity using the AA criterion. G.SRT.E.3.BA

BeB. May be able to, given two similar triangles identify the two pairs of angles that are congruent. G.SRT.E.3.BEB

F. Prove theorems involving similarity. G.SRT.F

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. G.SRT.F.4

Ad. Prove theorems about other regular figures by making generalizations of triangle proofs and theorems. G.SRT.F.4.AD

P. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. G.SRT.F.4.P

Ba. Informally prove, (including by measurement or inspection) theorems about triangles, and:
Complete proofs of theorems about triangles. AND/OR
Order the steps of a proof of theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. G.SRT.F.4.BA

BeB. May be able to, with assistance:
Informally prove, (including by measurement or inspection) theorems about triangles. OR
Complete proofs of theorems about triangles. OR
Order the steps of a proof of theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. G.SRT.F.4.BEB

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. G.SRT.F.5

Ad. Use congruence and similarity criteria for triangles to solve problems and explain why a geometric figure has been incorrectly deemed congruent or similar. G.SRT.F.5.AD

P. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. G.SRT.F.5.P

Ba. Use congruence or similarity criteria for triangles to solve problems or to prove relationships in geometric figures. G.SRT.F.5.BA

BeB. May be able to, when given specific information about similarity or congruent triangles, solve problems or identify the given relationship for the triangles. G.SRT.F.5.BEB

G. Define trigonometric ratios and solve problems involving right triangles. G.SRT.G

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.G.6

Ad. In addition to Proficient, the Advanced student is able to using data taken from several pairs of similar right triangles, establish generalities about the sides of right triangles and their relationship to the acute angles of said triangles. G.SRT.G.6.AD

P. The Proficient student is able to understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.G.6.P

Ba. The Basic student is able to demonstrate the ability to correctly orient two similar right triangles in order to identify the relationship between sides and angles. G.SRT.G.6.BA

BeB. The Below Basic student may be able to identify (verbally or in writing) the relationship between sides and angles when given two similar right triangles, each similarly oriented. G.SRT.G.6.BEB

7. Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.G.7

Ad. In addition to Proficient, the Advanced student is able to use data about side lengths of several right triangles and the properties of similar triangles, derive generalities about the sine and cosine relationships found. G.SRT.G.7.AD

P. The Proficient student is able to explain and use the relationship between the sine and cosine of complementary angles. G.SRT.G.7.P

Ba. The Basic student is able to use the relationship between the sine and cosine of complementary angles. G.SRT.G.7.BA

BeB. The Below Basic student may be able to use the sine and cosine relationships with assistance. G.SRT.G.7.BEB

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G.SRT.G.8

Ad. In addition to Proficient, the Advanced student is able to use trigonometric ratios and the Pythagorean Theorem to solve right triangles and verify proposed solutions in applied problems. G.SRT.G.8.AD

P. The Proficient student is able to use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G.SRT.G.8.P

Ba. The Basic student is able to solve right triangles in applied problems when given the trigonometric ratios and the Pythagorean Theorem. G.SRT.G.8.BA

BeB. The Below Basic student may be able to identify parts of a given right triangle figure that correspond to an applied problem for use in the formula when given trigonometric ratios and/or the Pythagorean Theorem. G.SRT.G.8.BEB

H. Apply trigonometry to general triangles. G.SRT.H

9. (+) Derive the formula A = ½ab sin(c) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.H.9

Ad. (+) The Advanced student is able to:
Explain if the formula, A = ½ab sin(C) would be appropriate for use in a compound figure and if possible, use the formula to determine the area of the compound figure. OR
Use the area formula in a novel way. G.SRT.H.9.AD

P. (+) Derive the formula A = ½ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.H.9.P

Ba. (+) Complete the derivation of the formula A = ½ab sin(C) by choosing appropriate steps from a provided list or correctly ordering the steps of a completed derivation. G.SRT.H.9.BA

BeB. (+) May be able to use A = ½ab sin(C) to find the area of a triangle when provided the formula and a labeled figure. G.SRT.H.9.BEB

10. (+)Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.H.10

Ad. (+) The Advanced student is able to:
Demonstrate instances when using the Law of Sines or the Law of Cosines would not be appropriate. OR Detect errors in the work of others. G.SRT.H.10.AD

P. (+) Prove the Law of Sines and the Law of Cosines, and use them to solve problems. G.SRT.H.10.P

Ba. (+) Use the Law of Sines and the Law of Cosines to solve problems when provided with the laws and complete proofs of the Law of Sines and the Law of Cosines by: Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.SRT.H.10.BA

BeB. (+) May be able to use the Law of Sines and the Law of Cosines to solve problems when provided with the laws or complete proofs of the Law of Sines and the Law of Cosines by:
Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.SRT.H.10.BEB

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). G.SRT.H.11

Ad. (+) The Advanced student is able to:
Demonstrate instances when using the Law of Sines or Law of Cosines would not be appropriate. OR
Detect errors in the work of others. G.SRT.H.11.AD

P. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). G.SRT.H.11.P

Ba. (+) Use the Law of Sines and the Law of Cosines to solve problems when provided with the laws. G.SRT.H.11.BA

BeB. (+) May be able to use the Law of Sines and Law of Cosines to solve problems when provided with the laws when given assistance. G.SRT.H.11.BEB

Circles

I. Understand and apply theorems about circles. G.C.I

1. Prove that all circles are similar. G.C.I.1

Ad. The Advanced student is able to:
Extend the proof that all circles are similar to other appropriate curvilinear figures. OR
Detect errors in the work of others G.C.I.1.AD

P. Prove that all circles are similar. G.C.I.1.P

Ba. Explain the similar relationship between two circles. G.C.I.1.BA

BeB. May be able to recognize that two circles are similar and describe in simple terms the relationship verbally or in writing. G.C.I.1.BEB

2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G.C.I.2

Ad. Apply relationships among inscribed angles, radii, and chords to solve real-world problems. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G.C.I.2.AD

P. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G.C.I.2.P

Ba. Identify and describe some relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G.C.I.2.BA

BeB. May be able to identify or describe some relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G.C.I.2.BEB

3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G.C.I.3

Ad. The Advanced student is able to:
Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for regular polygons inscribed in a circle. OR
Solve real-world problems using the properties described above. G.C.I.3.AD

P. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G.C.I.3.P

Ba. Construct the inscribed and circumscribed circles of a triangle, and complete proofs of properties of angles for a quadrilateral inscribed in a circle by: Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.C.I.3.BA

BeB. May be able to construct the inscribed and circumscribed circles of a triangle, or complete proofs of properties of angles for a quadrilateral inscribed in a circle by:
Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.C.I.3.BEB

4. (+) Construct a tangent line from a point outside a given circle to the circle. G.C.I.4

Ad. (+) The Advanced student is able to:
Prove the construction of a tangent line from a point outside of a circle to a point of tangency is unique. OR Apply this concept to solve a real-world problem. G.C.I.4.AD

P. (+) Construct a tangent line from a point outside a given circle to the circle. G.C.I.4.P

Ba. (+) Construct a tangent line from a point outside a given circle to the circle with assistance. G.C.I.4.BA

BeB. (+) May be able to distinguish between tangent lines from a point outside a given circle to the circle, and lines from the same or a different point outside a given circle to the circle, that are not tangent to a circle (i.e., identify tangency). G.C.I.4.BEB

J. Find arc lengths and areas of sectors of circles. G.C.J

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. G.C.J.5

Ad. Find the constant of proportionality by comparing arc lengths and sector areas of angles with differing radii measurements. G.C.J.5.AD

P. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. G.C.J.5.P

Ba. Recognize a sector as a part of the whole measure of the area of a circle with the same radius. Find the area of that sector using the formula for the area of a sector and find the arc length of the segment created by the sector. G.C.J.5.BA

BeB. May be able to:
Recognize a sector as a part of the whole measure of the area of a circle with the same radius. OR
Find the area of that sector using the formula for the area of a sector and find the arc length of segment created by the sector. G.C.J.5.BEB

Expressing Geometric Properties With Equations

K. Translate between the geometric description and the equation for a conic section. G.GPE.K

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.K.1

Ad. Use algebraic techniques to draw connections between distance formula, Pythagorean Theorem, completing the square, and transformations of functions to write the equation of a circle and to develop logical arguments for the standard form of a circle. G.GPE.K.1.AD

P. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.K.1.P

Ba. Derive the equation of a circle of given center and radius using the Pythagorean Theorem or complete the square to find the center and radius of a circle given by an equation. G.GPE.K.1.BA

BeB. May be able to derive the equation of a circle of given center and radius using the Pythagorean Theorem with assistance or complete the square to find the center and radius of a circle given by an equation with assistance. G.GPE.K.1.BEB

2. (+) Derive the equation of a parabola given a focus and directrix. G.GPE.K.2

Ad. (+) Draw connections between standard form and conic form of parabola equations or detect errors in others' derivation of equations. G.GPE.K.2.AD

P. (+) Derive the equation of a parabola given a focus and directrix. G.GPE.K.2.P

Ba. (+) Derive the equation of a parabola given a focus and directrix by:
Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.GPE.K.2.BA

BeB. (+) May be able to, with assistance, derive the equation of a parabola given a focus and directrix by:
Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.GPE.K.2.BEB

3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G.GPE.K.3

Ad. (+) The Advanced student is able to:
Draw connections between the standard forms and the conic forms of ellipses and hyperbolas. OR
Detect errors in others' derivations of said equations. G.GPE.K.3.AD

P. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G.GPE.K.3.P

Ba. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant by:
Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.GPE.K.3.BA

BeB. (+) May be able to, with assistance, derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant by:
Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.GPE.K.3.BEB

L. Use coordinates to prove simple geometric theorems algebraically. G.GPE.L

4. Use coordinates to prove simple geometric theorems algebraically. G.GPE.L.4

Ad. The Advanced student is able to:
Use coordinates to prove complex geometric theorems. OR
Detect errors in the proofs of others. G.GPE.L.4.AD

P. Use coordinates to prove simple geometric theorems algebraically. G.GPE.L.4.P

Ba. Use coordinates to prove simple geometric theorems algebraically by:
Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.GPE.L.4.BA

BeB. May be able to, with assistance, use coordinates to prove simple geometric theorems algebraically by:
Choosing appropriate steps from a provided list. OR
Correctly ordering the steps of completed proofs. G.GPE.L.4.BEB

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.L.5

Ad. The Advanced student is able to:
Make observations and develop logical arguments about the relationship of lines found in real-world contexts (parallel and perpendicular).
Analyze and critique the work of others in using and proving slope criteria. G.GPE.L.5.AD

P. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.L.5.P

Ba. Informally prove the slope criteria for parallel or perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.L.5.BA

BeB. May be able to verify perpendicular and parallel lines when given different representations (e.g., graphs, tables, equations) and use them to solve geometric problems. G.GPE.L.5.BEB

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.L.6

Ad. Explore and draw conclusions about patterns found among directed line segments or geometric figures of different dimensions and different ratios (e.g., patterns in the coordinates, patterns in actual lengths, patterns when changing units). G.GPE.L.6.AD

P. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.L.6.P

Ba. Find the point on a directed line segment between two given points that partitions the segment in a given common unit ratio (e.g., 12, 14, 13). G.GPE.L.6.BA

BeB. May be able to given a directed line segment, identify the point between two given points for common parts and wholes in a whole ratio using unit ratios (e.g., 12, 14, 13). G.GPE.L.6.BEB

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula). G.GPE.L.7

Ad. Use coordinates to compute areas of polygons and verify the technique using alternative methods. G.GPE.L.7.AD

P. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula). G.GPE.L.7.P

Ba. Use coordinates to compute perimeters and areas of triangles and rectangles. G.GPE.L.7.BA

BeB. May be able to use coordinates to compute perimeters or areas of right triangles or rectangles. G.GPE.L.7.BEB

Geometric Measurement and Dimension

M. Explain volume formulas and use them to solve problems. G.GMD.M

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. G.GMD.M.1

Ad. The Advanced student is able to:
Critique the method and verify the logic used by others. OR
Use dissection arguments, Cavalieri's principle, and informal limit arguments to find the area or volume of real-world irregular figures (e.g., horseshoe, hand, putting green, area under a curve).
 G.GMD.M.1.AD

P. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. G.GMD.M.1.P

Ba. Given a figure, identify the unit shapes that could be used to determine the area and use the sum of the parts to determine a method that approximates the area. Given a three dimensional figure, identify the unit shapes that could be used to determine the volume and use the sum of the parts to determine a method that approximates the volume. G.GMD.M.1.BA

BeB. May be able to, given a figure, identify the unit shapes that could be used to determine the area. Given a three dimensional figure, identify the unit shapes that could be used to determine the volume. G.GMD.M.1.BEB

2. (+) Give an informal argument using Cavalieri's Principle for the formulas for the volume of a sphere and other solid figures. G.GMD.M.2

Ad. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures and explain their use in real-world situations. G.GMD.M.2.AD

P. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. G.GMD.M.2.P

Ba. (+) Given several sliced solid figures with dimensions labeled, give an informal argument to explain why some figures have equal volumes. G.GMD.M.2.BA

BeB. (+) May be able to, given several sliced solid figures with dimensions labeled, identify the figures that have equal volumes. G.GMD.M.2.BEB

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. G.GMD.M.3

Ad. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems with compound shapes built from cylinders, pyramids, cones, and/or spheres in real-world contexts. G.GMD.M.3.AD

P. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. G.GMD.M.3.P

Ba. Use given volume formulas and shapes with all of the dimensions labeled for cylinders, pyramids, cones, and spheres to solve problems. G.GMD.M.3.BA

BeB. May be able to use given volume formulas and shapes with all of the dimensions labeled for cylinders, pyramids, cones, or spheres to solve problems. G.GMD.M.3.BEB

Ma. Visualize relationships between two-dimensional and three-dimensional objects. G.GMD.MA

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional object. G.GMD.M.4

Ad. The Advanced student is able to:
Use an irregular two-dimensional slice and generate the three dimensional object created from rotating it about an axis. OR
Explore the slicing of commonly shaped solids and analyze the patterns you find when making different slices. OR
Show where to slice a cube or cylinder to get a minimum and maximum number of sides of the two dimensional cross-sections. G.GMD.M.4.AD

P. Identify the shapes of two dimensional cross-sections of three dimensional objects, and identify three dimensional objects generated by rotations of two-dimensional objects. G.GMD.M.4.P

Ba. Identify the shapes of two dimensional cross-sections of three dimensional objects, or identify three dimensional objects generated by rotations of two-dimensional objects. G.GMD.M.4.BA

BeB. May be able to identify the shapes of two dimensional cross-sections of three dimensional objects when sliced horizontally or vertically, or identifies three dimensional objects generated by rotations about a horizontal or vertical edge of two-dimensional objects. G.GMD.M.4.BEB

Modeling With Geometry

O. Apply geometric concepts in modeling situations. G.MG.O

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). G.MG.O.1

Ad. Combine three-dimensional shapes to create a real-world object and estimate the volume. G.MG.O.1.AD

P. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). G.MG.O.1.P

Ba. Choose the appropriate combination of geometric shapes to describe a specified object. G.MG.O.1.BA

BeB. May be able to choose the appropriate geometric shape to describe a specified object. G.MG.O.1.BEB

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). G.MG.O.2

Ad. Critique the reasoning of others/self and apply concepts of density (density = mass/volume) based on area and volume in modeling situations and compare results to real-world data, if available, to make adjustments to estimation methods. G.MG.O.2.AD

P. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). G.MG.O.2.P

Ba. Given two different unit rates of density based on area or volume, compare totals when given the overall area or volume (e.g., given persons per square mile estimate the total for each given area and compare them, given BTUs per cubic foot estimate the total for more than one larger volume and compare them). G.MG.O.2.BA

BeB. May be able to, given a unit rate of density based on area or volume, find an estimated total in a modeling situation (e.g., given persons per square mile estimate the total for a given area, given BTUs per cubic foot estimate the total for a larger volume). G.MG.O.2.BEB

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). G.MG.O.3

Ad. Design an object or structure to satisfy physical constraints and minimize cost and justify the reasoning. G.MG.O.3.AD

P. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). G.MG.O.3.P

Ba. Given volume and surface area formulas, (prisms, pyramids, and spheres) determine the amount of material required to create a structure with specific physical constraints. G.MG.O.3.BA

BeB. May be able to identify which geometric attribute(s) need(s) to be calculated and use given volume and surface area formulas, (prisms, pyramids, and spheres) to determine the amount of material required to create a structure with specific physical constraints. G.MG.O.3.BEB

Interpreting Categorical And Quantitative Data

A. **Summarize, represent, and interpret data on a single count or measurement variable.** S.ID.A

1. Represent data with plots on the real number line (dot plots, histograms, and box plots) by hand or using technology. S.ID.A.1

Ad. Compare and contrast the different data representations (dot plots, histograms, and box plots) to determine what information can be gleaned or lost from each and justify the most appropriate representation to use. S.ID.A.1.AD

P. Represent data with plots on the real number line (dot plots, histograms, and box plots) by hand or using technology. S.ID.A.1.P

Ba. Represent data with plots on the real number line (using dot plots, histograms, or box plots). S.ID.A.1.BA

BeB. May be able to represent data with plots on the real number line (using a specified representation: dot plots and/or histograms and/or box plots) and given a pictorial or verbal example of each type that is to be created. S.ID.A.1.BEB

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.A.2

Ad. Find the appropriate visual representation and justify the appropriate measure of center and spread, given two or more sets of data. S.ID.A.2.AD

P. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.A.2.P

Ba. Inconsistently able to use appropriate terminology to describe similarities and differences when comparing center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.A.2.BA

BeB. May be able to informally compare the similarities and differences when comparing center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.A.2.BEB

3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). S.ID.A.3

Ad. In addition to Proficient, the Advanced student is able to given different shapes, context, and statistics for sets of data, discern and predict the differences caused by omission or inclusion of extreme data points (outliers) in data sets, including those with uniform or near uniform values. S.ID.A.3.AD

P. The Proficient student is able to interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). S.ID.A.3.P

Ba. The Basic student is able to interpret differences in any two of the following: shape, center, or spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). S.ID.A.3.BA

BeB. The Below Basic student may be able to when comparing different data sets using the same representation, Identify the impact of extreme data points (outliers) on the shape (skewed, symmetrical, or constant), center (mean or median), or spread (interquartile range or standard deviation). S.ID.A.3.BEB

4. (+) Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use the Empirical Rule, calculators, spreadsheets, and/or tables to estimate areas under the normal curve. S.ID.A.4

Ad. (+) Using the mean, standard deviation, other statistics, and visual representations of several data sets, compare the visual representations to determine and justify the appropriateness of fitting to a normal curve to estimate population percentages. S.ID.A.4.AD

P. (+) Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use the Empirical Rule, calculators, spreadsheets, and/or tables to estimate areas under the normal curve. S.ID.A.4.P

Ba. (+) Use the mean and standard deviation of a data set to attempt to fit it to a normal distribution and to estimate population percentages, not recognizing when such procedures might not be appropriate. Use the Empirical Rule, calculators, spreadsheets, and/or tables to make population estimates. S.ID.A.4.BA

BeB. (+) May be able to, given a normal distribution, estimate population percentages using the Empirical Rule, calculators, spreadsheets, and/or tables. S.ID.A.4.BEB

5. (+) Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations in the data, and use inferential statistical techniques to show association. S.ID.B.5

Ad. (+) Identify real-world problems where chi-square analysis (i.e., goodness of fit, homogeneity, and independence) would be appropriate and use chi-square analysis to solve these problems. S.ID.B.5.AD

P. (+) Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations in the data, and use inferential statistical techniques to show association. S.ID.B.5.P

Ba. (+) Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and/or conditional relative frequencies). S.ID.B.5.BA

BeB. (+) Does not meet the basic performance level. S.ID.B.5.BEB

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S.ID.B.6

A. Use a function to describe data trends to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. S.ID.B.6.A

B. (+) Informally assess the fit of a function by plotting and analyzing residuals. S.ID.B.6.B

C. Using technology, fit a least squares linear regression function for a scatter plot that suggests a linear association. S.ID.B.6.C

Ad. The Advanced student is able to:
Determine the best model and justify by describing the pros and cons of the data representation for two quantitative variables on a scatter plot using alternative linear, quadratic, and exponential models. AND
Determine the effect of removing extreme values (outliers) on the model. Make an argument for or against removing extreme values (outliers). AND
Discuss appropriate use of the model to make predictions while attending to precision (correct data entry errors). S.ID.B.6.AD

P. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S.ID.B.6.P

A. Use a function to describe data trends to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. S.ID.B.6.P.A

B. (+) Informally assess the fit of a function by plotting and analyzing residuals. S.ID.B.6.P.B

C. Using technology, fit a least squares linear regression function for a scatter plot that suggests a linear association. S.ID.B.6.P.C

Ba. The Basic student is able to:
Given two models (linear, quadratic, or exponential) determine which model best represents the data by informally assessing the fit of a function by plotting and/or analyzing residuals. OR
Given a data set, use technology to create a scatter plot and the least squares regression function. S.ID.B.6.BA

BeB. May be able to represent data on two quantitative variables on a scatter plot and informally determine if a linear, quadratic, or exponential is a best fit. S.ID.B.6.BEB

C. Interpret linear models. S.ID.C

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.C.7

Ad. The Advanced student is able to:
Make predictions using the rate of change and the constant term of a linear model in the context of the data. Determine and explain when extrapolation is appropriate or inappropriate. OR
Identify a data source, formulate questions about the data, and explain in context how the slope and intercept would be interpreted. S.ID.C.7.AD

P. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.C.7.P

Ba. Determine the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.C.7.BA

BeB. May be able to, given the slope (rate of change) and the intercept (constant term) of a linear model, locate these in a scatter plot of the same data. S.ID.C.7.BEB

8. Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.C.8

Ad. Compare different scatter plots with the same correlation coefficient to determine if a linear model best fits the data by examining shape and statistics and justify reasoning. S.ID.C.8.AD

P. Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.C.8.P

Ba. Compute (using technology) or interpret the correlation coefficient of a linear fit. S.ID.C.8.BA

BeB. May be able to compute (using technology) or interpret the correlation coefficient of a linear fit with the assistance of written or pictorial guided steps or video. S.ID.C.8.BEB

9. Distinguish between correlation and causation. S.ID.C.9

Ad. The Advanced student is able to:
Support or refute claims of causation from a real-world example (e.g., newspaper, website) with the understanding that a strong correlation does not imply causation. OR
Research or create two sets of data and their context to demonstrate how correlation and causation could be confused. Explain the reasons for confusion. S.ID.C.9.AD

P. Distinguish between correlation and causation. S.ID.C.9.P

Ba. Identify the existence or nonexistence of causation in the context of a correlated problem. S.ID.C.9.BA

BeB. May be able to, when provided with two disparate examples, determine which demonstrates correlation. S.ID.C.9.BEB

Making Inferences And Justifying Conclusions

D. Understand and evaluate random processes underlying statistical experiments. S.IC.D

1. (+) Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.D.1

Ad. (+) Select a random sample from a real-world population and use statistics to make appropriate inferences. S.IC.D.1.AD

P. (+) Use statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.D.1.P

Ba. (+) Draw logical conclusions about a population when provided with descriptive statistics from a random sample from that population. S.IC.D.1.BA

BeB. (+) Does not meet the basic performance level. S.IC.D.1.BEB

2. (+) Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. S.IC.D.2

Ad. (+) Generate or estimate a model consistent with results from a given data-generating process, e.g., using simulation. S.IC.D.2.AD

P. (+) Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. S.IC.D.2.P

Ba. (+)Match a plot for data from a specified real-world situation with a given model. S.IC.D.2.BA

BeB. (+) Does not meet the basic performance level. S.IC.D.2.BEB

E. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. S.IC.E

3. (+) Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.E.3

Ad. (+) Draw multiple random samples to complete a survey, experiment, or observational study. Compare and discuss the results. S.IC.E.3.AD

P. (+) Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.E.3.P

Ba. (+) Explain how results can be biased if the sample is not randomly selected, e.g., a convenience sample vs. a random sample. S.IC.E.3.BA

BeB. (+) Does not meet the basic performance level. S.IC.E.3.BEB

4. (+) Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.E.4

Ad. (+) Develop a confidence interval for the population mean or proportion using the data from the sample survey and relate it to the margin of error from the simulation. S.IC.E.4.AD

P. (+) Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.E.4.P

Ba. (+) Use data from a sample survey to estimate a population mean or proportion and use the formula to calculate the margin of error. S.IC.E.4.BA

BeB. (+) Does not meet the basic performance level. S.IC.E.4.BEB

5. (+) Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.E.5

Ad. (+) Do a statistical analysis (i.e., t-test) of the data from a randomized experiment to compare two treatments. Report results to determine if the differences between the parameters are significant. S.IC.E.5.AD

P. (+) Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.E.5.P

Ba. (+) Given a plot comparing the two treatments from a randomized experiment, construct a logical argument about whether or not the parameters (proportion or mean) would be different. S.IC.E.5.BA

BeB. (+) Does not meet the basic performance level. S.IC.E.5.BEB

6. (+) Evaluate reports based on data. S.IC.E.6

Ad. (+) Evaluate a report discussing the sampling technique, the data collection instruments, the assumptions of the statistical analysis used, the data analysis, and the accuracy of the conclusions drawn. S.IC.E.6.AD

P. (+) Able to (+) evaluate reports based on data. S.IC.E.6.P

Ba. (+) Evaluate reports by identifying the type of sampling done and comment on its appropriateness. Discuss if there is data provided to support the conclusions. S.IC.E.6.BA

BeB. (+) Does not meet the basic performance level. S.IC.E.6.BEB

Conditional Probability And The Rules Of

F. **Understand independence and conditional probability and use them to interpret data.** S.CP.F

Probability

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.F.1

Ad. The Advanced student is able to:
Compare at least two different representations (e.g., Venn Diagram, two-way table, set notation, verbal description) of events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). OR
Develop questions that can be answered using unions, intersections, or complements. S.CP.F.1.AD

P. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.F.1.P

Ba. Describe events as subsets using three of the four characteristics: Outcomes.
Unions.
Intersection.
Complements. S.CP.F.1.BA

BeB. May be able to describe events as subsets using two of the four characteristics: Outcomes.
Unions.
Intersection.
Complements. S.CP.F.1.BEB

2. (+) Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.F.2

Ad. (+) Identify real-world situations where P(A) and P(B) can be used to determine if the events A and B are independent by deriving the probabilities P(A), P(B), P(A and B) and interpret results. S.CP.F.2.AD

P. (+) Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.F.2.P

Ba. (+) Determine if events A and B are independent when given probabilities P(A), P(B), and P(A and B). S.CP.F.2.BA

BeB. (+) May be able to does not meet the basic performance level. S.CP.F.2.BEB

3. (+) Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A and B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. S.CP.F.3

Ad. (+) Identify real-world situations where P(A) ad P(B) can be used to determine if the events A and B are independent by deriving the probabilities P(A), P(B), P(A/B), P(B/A), and P(A and B) and interpret results. S.CP.F.3.AD

P. (+) Is able to (+) understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. S.CP.F.3.P

Ba. (+) Determine if events A & B are independent when given probabilities P(A), P(B), P(A/B), P(B/A), and P(A and B). S.CP.F.3.BA

BeB. (+) Does not meet the basic performance level. S.CP.F.3.BEB

4. (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. S.CP.F.4

Ad. (+) Identify a real-world situation and collect data that is appropriate for constructing a two-way frequency table. Construct and interpret the two-way frequency table of data when two categories are associated with each object being classified. Analyze and describe the results. S.CP.F.4.AD

P. (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. S.CP.F.4.P

Ba. (+) Interpret a given two-way frequency table of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. S.CP.F.4.BA

BeB. (+) Does not meet the basic performance level. S.CP.F.4.BEB

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. S.CP.F.5

Ad. Identify a real-world situation that uses independence and identify a real-world situation that uses dependence. S.CP.F.5.AD

P. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. S.CP.F.5.P

Ba. Recognize the concepts of conditional probability or independence in everyday language and everyday situations. S.CP.F.5.BA

BeB. May be able to, with assistance, recognize the concepts of conditional probability or independence in everyday language and everyday situations. S.CP.F.5.BEB

G. Use the rules of probability to compute probabilities of compound events in a uniform probability model. S.CP.G

6. (+) Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.G.6

Ad. (+) In addition to Proficient, the Advanced student is able to explain why P(A/B) is different than P(B/A) in a real-world situation. S.CP.G.6.AD

P. (+) The Proficient student is able to find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.G.6.P

Ba. (+) The Basic student is able to calculate conditional probability of A given B when given probabilities P(A), A(B), P(A/B), P(B/A), and P(A and B). S.CP.G.6.BA

BeB. (+) The Below Basic student does not meet the basic performance level. S.CP.G.6.BEB

7. (+) Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. S.CP.G.7

Ad. (+) Identify real-word situations where the addition rule would apply, derive the appropriate probabilities, solve the problem, and interpret the results. S.CP.G.7.AD

P. (+) Apply the addition rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. S.CP.G.7.P

Ba. (+) Apply the addition rule and interpret results when given probabilities P(A), P(B), and P(A and B). S.CP.G.7.BA

BeB. (+) Does not meet the basic performance level. S.CP.G.7.BEB

8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B/A)] = [P(B)]x[P(A/B)], and interpret the answer in terms of the model. S.CP.G.8

Ad. (+) Identify real-word situations where the multiplication rule would apply, derive the appropriate probabilities, solve the problem and interpret the results. S.CP.G.8.AD

P. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B/A)] = [P(B)]x[P(A/B)], and interpret the answer in terms of the model. S.CP.G.8.P

Ba. (+) Apply the multiplication rule and interpret results when given probabilities P(A), P(B), P(A/B), and P(B/A). S.CP.G.8.BA

BeB. (+) Does not meet the basic performance level. S.CP.G.8.BEB

9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. S.CP.G.9

Ad. (+) Identify a real-world problem that requires permutations and combinations to compute the probability of a compound event, derive the required values, solve the problem, and interpret the results. S.CP.G.9.AD

P. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. S.CP.G.9.P

Ba. (+) Determine the probability of a compound event when given the values of the permutations and the combinations applicable to the problem. S.CP.G.9.BA

BeB. (+) Does not meet the basic performance level. S.CP.G.9.BEB

Use Probability To Make Decisions

H. Calculate expected values and use them to solve problems. S.MD.H

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.H.1

Ad. (+) Construct a statistical experiment by identifying an appropriate random variable, listing the events in the sample space, calculating the probability distribution, and constructing the appropriate graphical display. Interpret the results. S.MD.H.1.AD

P. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.H.1.P

Ba. (+) Given a random variable for a quantity of interest and the numerical value for each event in the sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.H.1.BA

BeB. (+) Does not meet the basic performance level. S.MD.H.1.BEB

2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. S.MD.H.2

Ad. (+) Gather data for a real-world situation where expected value could be used to make an advantageous decision (e.g., lottery ticket, card game, dice game, investments). Calculate the expected value and explain or support the decision. S.MD.H.2.AD

P. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. S.MD.H.2.P

Ba. (+) Calculate expected value when given the formula E(X) = Σ(X_i * P(X_i)) and a table with X_i and P(X_i). S.MD.H.2.BA

BeB. (+) Does not meet the basic performance level. S.MD.H.2.BEB

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. S.MD.H.3

Ad. (+) Construct a statistical experiment with a random variable and develop a table of X_i, P(X_i) values, and determine the expected value. Interpret the results. S.MD.H.3.AD

P. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. S.MD.H.3.P

Ba. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated. S.MD.H.3.BA

BeB. (+) Does not meet the basic performance level. S.MD.H.3.BEB

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. S.MD.H.4

Ad. (+) Simulate a random process by defining the sample space and developing the probability distribution for a random variable. Calculate the expected value and interpret the results. S.MD.H.4.AD

P. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. S.MD.H.4.P

Ba. (+) Complete a probability distribution for a random variable from a simple experiment (e.g., a coin flip, tossing a die, drawing a card) defined for a sample space in which probabilities are assigned empirically. S.MD.H.4.BA

BeB. (+) Does not meet the basic performance level. S.MD.H.4.BEB

I. Use probability to evaluate outcomes of decisions. S.MD.I

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. S.MD.I.5

A. Find the expected payoff for a game of chance. S.MD.I.5.A

B. Evaluate and compare strategies on the basis of expected values. S.MD.I.5.B

Ad. (+) Gather data to make the advantageous decision for a real-world situation. Explain reasoning for the decision. S.MD.I.5.AD

P. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. S.MD.I.5.P

A. Find the expected payoff for a game of chance. S.MD.I.5.P.A

B. Evaluate and compare strategies on the basis of expected values. S.MD.I.5.P.B

Ba. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and/or finding expected values. Find the expected payoff for a game of chance. S.MD.I.5.BA

BeB. (+) Does not meet the basic performance level. S.MD.I.5.BEB

J. Calculate expected values and use them to solve problems. S.MD.J

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.J.6

Ad. (+) Identify real-world situations where probability could be used to make fair and unfair decisions. Explain the reasoning. S.MD.J.6.AD

P. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.J.6.P

Ba. (+) Compare given probability distributions. State which distribution would result in a fair decision. S.MD.J.6.BA

BeB. (+) Does not meet the basic performance level. S.MD.J.6.BEB

7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). S.MD.J.7

Ad. (+) Analyze decisions and strategies using probability concepts, identify the advantages and disadvantages of the possible decisions, and justify the best choice (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). S.MD.J.7.AD

P. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). S.MD.J.7.P

Ba. (+) Analyze decisions and strategies using probability concepts when given the probabilities (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). S.MD.J.7.BA

BeB. (+) Does not meet the basic performance level. S.MD.J.7.BEB